Centers and invariant lines of planar polynomial differential systems and its configurations

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Abstract

In this talk, we will give the least upper bound on the number of centers of all planar polynomial Hamiltonian vector fields. This least upper bound characterizes the relationship between the number of centers and the number of critical points at infinity, which generalizes the nice result by Cima, Gasull and Mañosas in [JDE, 1993] and reveals that the greater the number of invariant lines of the vector field and the less the number of centers. Furthermore, we will consider planar polynomial Hamiltonian vector fields with two intersecting invariant straight lines, and obtain some rules on the configurations of its centers when the number of its centers is exactly the least upper bound. As an application of these results, we will prove that there are only two kinds of global phase portraits for the cubic polynomial Hamiltonian vector fields with two intersecting invariant lines having four centers, which solves the open problem proposed by Llibre and Xiao in [PJM, 2020]. Interestingly, a cubic polynomial vector field with two invariant lines must have elementary functional first integrals if it has four centers. It is shown that this cubic polynomial vector fields with four centers has and only has three different kinds of configurations. In other words, non-Hamiltonian cubic polynomial vector fields with four centers can have more configurations of centers than that of Hamiltonian ones. This talk is based on a joint work with Hongjin He and Changjian Liu.