

Alexey Balitskiy, IAS/Princeton

Title: Systolic freedom and rigidity modulo 2

The k -dimensional systole of an n -dimensional closed Riemannian manifold M is the infimal k -volume of a non-trivial k -cycle (with some coefficients). In '90s, Gromov asked if the product of the k -systole and the $(n - k)$ -systole is bounded from above by the volume of M (up to a dimensional factor); this would manifest the *systolic rigidity*. Freedman exhibited the first examples with $k = 1$ and mod 2 coefficients where this fails; this manifests the *systolic freedom*. In a joint work with Hannah Alpert and Larry Guth, we showed that Freedman's examples are almost as "free" as possible, and the systolic rigidity almost holds, with $k = 1$ and mod 2 coefficients. Namely, on a manifold of bounded local geometry, $\text{systole}_1(M) \cdot \text{systole}_{n-1}(M) \leq c_\epsilon \text{volume}(M)^{1+\epsilon}$, as long as the left-hand side is finite ($H_1(M; \mathbb{Z}/2)$ is non-trivial). The proof, which I will explain, is based on the Schoen–Yau–Guth–Papasoglu minimal surface method.