Multivariate Stochastic Volatility Models and Large Deviation Principles

Archil Gulisashvili

Ohio University

Stochastic Analysis and Stochastic Partial Differential Equations

A Celebration of Marta Sanz-Solé's Mathematical Legacy

Centre de Recerca Matematica, Barcelona, May 30 -- June 3, 2022

Outline

The following topics will be addressed in the talk:

- Multivariate time-inhomogeneous stochastic volatility models.
- Volterra type volatility processes.
- Restrictions on volatility models.
- Sample path large deviation principles for log-processes.
- Large deviation principles for volatility processes.
- First exit times for log-processes.
- Binary barrier options.
- Examples of models satisfying a comprehensive large deviation principle.

The talk is based on the paper: A. G., Multivariate stochastic volatility models and large deviation principles, submitted for publication, available at arXiv:2203.09015, 2022 (see also the author's papers [11] – [15]).

More papers to pay attention to: Bayer-Friz-Gassiat-Martin-Stemper [2], Chiarini-Fischer [5], Jacquier-Pannier [16], Nualart-Rovira [18], Rovira–Sanz-Solé [20], Wang [22], Zhang [23].

Multivariate Time-Inhomogeneous Stochastic Volatility Models

This talk deals with general multivariate time-inhomogeneous stochastic volatility models. Such a model is described by the following multidimensional stochastic differential equation:

$$dS_t = S_t \circ [b(t, \widehat{B}_t)dt + \sigma(t, \widehat{B}_t)(\bar{C}dW_t + CdB_t)], \quad 0 \leq t \leq T, \quad S_0 = s_0 \in \mathbb{R}^m \tag{1}$$

where the initial condition $s_0 = (s_0^{(1)}, \cdots, s_0^{(m)})$ is such that $s_i > 0$ for all $1 \leq i \leq m$.

- The equation in (1) is defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ carrying two independent m-dimensional standard Brownian motions W and B with respect to the measure $\mathbb{P}$.

- By $\{\mathcal{F}_t\}_{0\le t\le T}$ is denoted the augmentation of the filtration generated by the processes W and B. We will also use the augmentation of the filtration generated by the process B, and denote it by $\{\mathcal{F}_t^B\}_{0\le t\le T}$.

- The symbol b in (1) stands for a continuous map defined on $[0, T] \times \mathbb{R}^d$ with values in $\mathbb{R}^m$. We call b the drift map. By σ is denoted a continuous map of $[0, T] \times \mathbb{R}^d$ into the space of $(m \times m)$ real matrices. This map is called the volatility map.

- The process $\widehat{B} = (\widehat{B}^{(1)}, \cdots, \widehat{B}^{(d)})$ appearing in (1) is a continuous d-dimensional stochastic process adapted to the filtration $\{\mathcal{F}_t^B\}_{0\le t\le T}$. The process $\widehat{B}$ is called the volatility process.

- $\mathbb{R}^m$ is the m-dimensional Euclidean space equipped with the norm $||\cdot||_m$.
- For a real $(m \times m)$-matrix M, its Frobenius norm will be denoted by $||M||_{m\times m}$ and the symbol M' stands for the transpose of M.
- The symbol $\circ$ in (1) stands for the Hadamard (component-wise) product of vectors.
- The matrix C in (1) is a real $(m \times m)$-matrix such that $||C||_{m\times m} < 1$. It is clear that the matrix $\mathrm{Id_m} - \mathrm{C'C}$ is symmetric and positive definite, and we denote the unique symmetric and positive definite square root of the matrix $\mathrm{Id_m} - \mathrm{C'C}$ by $\bar{C}$.
- The model in (1) can be interpreted as a time-inhomogeneous stochastic volatility model describing the time-behavior of price processes of correlated risky assets. The matrix-valued process $\sigma(t, \widehat{B}_t)$, with $t \in [0, T]$, characterizes the joint volatility of these assets.

Set-Ups. Canonical set-up on $\mathcal{W}^p$

We adopt the terminology (set-ups) used in Rogers-Williams [19].

Definition 1. *The system* $(\Omega, W, B, \mathcal{F}_T, \{\mathcal{F}_t\}_{0\le t\le T}, \mathbb{P})$ *is called a set-up associated with the model in (1), while the system* $(\Omega, B, \mathcal{F}_T^B, \{\mathcal{F}_t^B\}_{0\le t\le T}, \mathbb{P})$ *is called a set-up associated with the volatility process in (1).*

Canonical set-up. For a positive integer $p \ge 1$, the symbol $\mathcal{W}^p$ stands for the space of continuous $\mathbb{R}^p$-valued maps on $[0,T]$ equipped with the norm $||f|| = \max_{t\in[0,T]} ||f(t)||_p$, $f \in \mathcal{W}^p$.

Let B_s, with $s \in [0,T]$, be the coordinate process on $\mathcal{W}^p$. Define a filtration on the space $\mathcal{W}^p$ by $\mathcal{B}_t^p = \sigma(B_s : 0 \le s \le t)$, $t \in [0,T]$. The augmentation $\{\widetilde{\mathcal{B}}_t^p\}$ of the filtration $\{\mathcal{B}_t^p\}$ is called the canonical filtration on $\mathcal{W}^p$.

Let $\mathbb{P}$ be the Wiener measure on $\widetilde{\mathcal{B}}_T^p$. The coordinate process $s \mapsto B_s$ plays the role of p-dimensional standard Brownian motion with respect to the measure $\mathbb{P}$.

Definition 2. *The ordered system* $(\mathcal{W}^p, B, \widetilde{\mathcal{B}}_T^p, \{\widetilde{\mathcal{B}}_t^p\}, \mathbb{P})$ *is called the canonical set-up on* $\mathcal{W}^p$.

Canonical Set-Up on $\mathcal{W}^m \times \mathcal{W}^m$

- Denote the coordinate processes on Ω_1 and Ω_2 by W and B, respectively, and consider the filtration on Ω generated by the process $t \mapsto (W_t, B_t)$, $t \in [0, T]$.

- Denote by $\{\mathcal{F}_t\}$ the augmentation of this filtration with respect to the measure $\mathbb{P} = \mathbb{P}_1 \times \mathbb{P}_2$, where $\mathbb{P}_1$ and $\mathbb{P}_2$ are the Wiener measures on Ω_1 and Ω_2, respectively.

- By $\{\mathcal{F}_t^B\}$ will be denoted the augmentation of the filtration generated by process $t \mapsto B_t$, $t \in [0, T]$.

- The processes W and B are independent m-dimensional Brownian motions defined on the space Ω.

- The system $(\Omega, W, B, \mathcal{F}_T, \{\mathcal{F}_t\}, \{\mathcal{F}_t^B\}, \mathbb{P})$ is called the canonical set-up on the space $\Omega = \mathcal{W}^m \times \mathcal{W}^m$.

The Log-Process

It can be established using the Doléans-Dade formula that the log-process $X = \log S$ can be represented as follows:

$$X_t = x_0 + \int_0^t b(s, \widehat{B}_s)ds - \frac{1}{2}\int_0^t \mathrm{diag}(\sigma(s, \widehat{B}_s)\sigma(s, \widehat{B}_s)')ds$$
$$+ \int_0^t \sigma(s, \widehat{B}_s)(\bar{C}dW_s + CdB_s), \quad 0 \le t \le T.$$

Let $\varepsilon \in (0, 1]$ be the scaling parameter. The scaled version of the log-process X is defined by

$$X_t^{(\varepsilon)} = x_0 + \int_0^t b(s, \widehat{B}_s^{(\varepsilon)})ds - \frac{1}{2}\varepsilon\int_0^t \mathrm{diag}(\sigma(s, \widehat{B}_s^{(\varepsilon)})\sigma(s, \widehat{B}_s^{(\varepsilon)})')ds$$
$$+ \sqrt{\varepsilon}\int_0^t \sigma(s, \widehat{B}_s^{(\varepsilon)})(\bar{C}dW_s + CdB_s)$$

where $X_0^{(\varepsilon)} = x_0$ for all $s \in (0, 1]$. The scaled volatility process $\widehat{B}^{(\varepsilon)}$ appearing in the previous formula will be introduced later.

One-Factor Models

In the case where $m = 1$, we use the correlation parameter $\rho \in (-1, 1)$ and set $\bar{\rho} = \sqrt{1 - \rho^2}$.

Therefore, the equation describing the evolution of the process S is as follows:

$$dS_t = S_t[b(t, \widehat{B}_t)dt + \sigma(t, \widehat{B}_t)(\bar{\rho}dW_t + \rho dB_t)], \quad S_0 = s_0 > 0.$$

Moreover, the log-process is given by

$$X_t = x_0 + \int_0^t b(s, \widehat{B}_s)ds - \frac{1}{2}\int_0^t \sigma(s, \widehat{B}_s)^2 ds + \int_0^t \sigma(s, \widehat{B}_s)(\bar{\rho}dW_s + \rho dB_s)$$

where $x_0 = \log s_0$.

Assumption A

A modulus of continuity is a nonnegative nondecreasing funciton ω on $[0,\infty)$ such that $\omega(s) \to 0$ as $s \to 0$.

Let $x = (t_1, v_1)$ and $y = (t_2, v_2)$ be elements of the space $[0,T] \times \mathbb{R}^d$ equipped with the Euclidean distance $\nu_d(x,y) = \sqrt{(t_1 - t_2)^2 + ||v_1 - v_2||_d^2}$. Denote by $\overline{B_d(r)}$ the closed ball centered at $(0,0)$ of radius $r > 0$ in the metric space defined above, and let ω be a modulus of continuity on $[0,\infty)$.

Definition 3. *A map $\lambda : [0,T] \times \mathbb{R}^d \mapsto \mathbb{R}^1$ is called locally ω-continuous if for every $r > 0$ there exists $L(r) > 0$ such that for all $x, y \in \overline{B_d(r)}$ the following inequality holds:*

$$|\lambda(x) - \lambda(y)| \leq L(r)\omega(\nu_d(x,y)).$$

Assumption A: The components of the drift map b and the elements of the volatility map σ are locally ω-continuous on the space $[0,T] \times \mathbb{R}^d$ for some modulus of continuity ω. In addition, the elements of the volatility map σ are not identically zero on $[0,T] \times \mathbb{R}^d$.

Volatility Equations

Let Y be a stochastic process satisfying the following Volterra type stochastic integral equation on $\mathcal{W}^m$ equipped with the canonical set-up:

$$Y_t = y + \int_0^t a(t,s,V^{(1)},Y)ds + \int_0^t c(t,s,V^{(2)},Y)dB_s.$$

- The previous equation will be called the volatility equation. In it, a is a map from the space $[0,T]^2 \times \mathcal{W}^{k_1} \times \mathcal{W}^d$ into the space $\mathbb{R}^d$, while c is a map from the space $[0,T]^2 \times \mathcal{W}^{k_2} \times \mathcal{W}^d$ into the space of $(d \times m)$-matrices. More restrictions on the maps a and c will be introduced later.

- The processes $V^{(i)}$, $i = 1,2$, appearing in the volatility equation, are fixed auxiliary continuous stochastic processes on $\mathcal{W}^m$ with state spaces $\mathbb{R}^{k_1}$ and $\mathbb{R}^{k_2}$, respectively.

- These processes satisfy the following stochastic differential equations:

$$V_s^{(i)} = V_0^{(i)} + \int_0^s \bar{b}_i(r,V^{(i)})dr + \int_0^s \bar{\sigma}_i(r,V^{(i)})dB_r, \quad i = 1,2 \tag{2}$$

  where $V_0^{(i)} \in \mathbb{R}^{k_i}$ are initial conditions, $\bar{b}_i$ are maps of $[0,T] \times \mathcal{W}^{k_i}$ into $\mathbb{R}^{k_i}$, while $\bar{\sigma}_i$ are maps of $[0,T] \times \mathcal{W}^{k_i}$ into the space of $k_i \times m$-matrices.

It is assumed that the equations in (2) satisfy Conditions (H1) - (H6) introduced in [5] by Chiarini and Fischer. This paper was an important source of ideas in our work on volatility processes. However, Chiarini and Fischer did not study Volterra type processes in the paper [5].

Remark 5. *Examples of equations for which Conditions (H1) - (H6) hold true, include equations with locally Lipschitz coefficients satisfying the sub-linear growth condition and one-dimensional diffusion equations with Hölder dispersion coefficient, e.g, the CIR-equation.*

Definition 6. *The volatility process $\widehat{B}$ used in the general stochastic volatility model has the following form: $\widehat{B} = GY$, where Y satisfies the volatility equation, while G is a continuous map from $\mathcal{W}^d$ into itself that is $\widetilde{\mathcal{B}}_t^d\,/\,\mathcal{B}_t^d$-measurable for every $t \in [0, T]$.*

Definition 7. *Denote by $(\mathbb{H}_0^1)^m$ the m-dimensional Cameron-Martin space. For every function $f \in (\mathbb{H}_0^1)^m$, the function $\widehat{f} \in \mathcal{W}^d$ is defined by $\widehat{f} = G(\Gamma_y \dot{f})$ where $\Gamma_y \dot{f}$ is the solution to the skeleton equation*

$$\Gamma_y \dot{f}(t) = y + \int_0^t a(t, s, \psi_{1,f}, \Gamma_y \dot{f}) ds + \int_0^t c(t, s, \psi_{2,f}, \Gamma_y \dot{f}) \dot{f}(s) ds$$

(see Definition 15).

Scaled Volatility Processes

A scaled version of the volatility equation has the following form:

$$Y_t^{(\varepsilon)} = y + \int_0^t a(t,s,V^{1,\varepsilon},Y^{(\varepsilon)})ds + \sqrt{\varepsilon}\int_0^t c(t,s,V^{2,\varepsilon},Y^{(\varepsilon)})dB_s.$$

- For every $i = 1,2$, the process $V^{i,\varepsilon}$ is a scaled version of the process $V^{(i)}$. It satisfies the equation

$$V_s^{i,\varepsilon} = V_0^{(i)} + \int_0^s \bar{b}_i(r,V^{i,\varepsilon})dr + \sqrt{\varepsilon}\int_0^s \bar{\sigma}_i(r,V^{i,\varepsilon})dB_r.$$

- The previous equation has the unique strong solution and the path independence holds this equation if the conditions in Remark 5 are satisfied.

Definition 7. *The scaled volatility process $\widehat{B}^{(\varepsilon)}$ is given by $\widehat{B}^{(\varepsilon)} = GY^{(\varepsilon)}$ where G is introduced in Definition 6, while $Y^{(\varepsilon)}$ is the solution to the scaled volatility equation.*

Sample Path LDP for the Log-Process

A sample path large deviation principle (LDP) for a stochastic process characterizes logarithmic asymptotics of the probability that the path of a scaled version of the process belongs to a given set of paths. The theory of sample path large deviations goes back to the celebrated work of Varadhan [21] and Freidlin and Wentzell [10].

Theorem 4. *Suppose Assumption A and Assumptions (C1) – (C7) hold true, and the model in (1) is defined on the canonical set-up. Then, the process* $\varepsilon \mapsto X^{(\varepsilon)} - x_0$ *with state space* $\mathcal{W}^m$ *satisfies the sample path large deviation principle with speed* ε^{-1} *and good rate function* $\widetilde{Q}_T$ *defined on the previous slide. The validity of the large deviation principle means that for every Borel measurable subset* $\mathcal{A}$ *of* $\mathcal{W}^m$*, the following estimates hold:*

$$
\begin{aligned}
&-\inf_{g\in\mathcal{A}^\circ}\widetilde{Q}_T(g) \le \liminf_{\varepsilon\downarrow 0}\varepsilon\log\mathbb{P}\left(X^{(\varepsilon)}-x_0\in\mathcal{A}\right)\\
&\le \limsup_{\varepsilon\downarrow 0}\varepsilon\log\mathbb{P}\left(X^{(\varepsilon)}-x_0\in\mathcal{A}\right) \le -\inf_{g\in\bar{\mathcal{A}}}\widetilde{Q}_T(g).
\end{aligned}
$$

The symbols $\mathcal{A}^\circ$ *and* $\bar{\mathcal{A}}$ *in the previous estimates stand for the interior and the closure of the set* $\mathcal{A}$*, respectively.*

The Rate Function

- The rate function $\widetilde{Q}_T$ governing the large deviation principle for the log-process depends on the measurable map $\Phi : \mathbb{C}_0^m \times \mathbb{C}_0^m \times \mathcal{W}^d \mapsto \mathbb{C}_0^m$ given by

$$\Phi(l,f,h)(t) = \int_0^t b(s,\widehat{f}(s))ds + \int_0^t \sigma(s,\widehat{f}(s))\bar{C}\dot{l}(s)ds + \int_0^t \sigma(s,\widehat{f}(s))C\dot{f}(s)ds$$

  for all $l, f \in (\mathbb{H}_0^1)^m$, $h = \widehat{f} \in \mathcal{W}^d$, and $0 \le t \le T$. For all the remaining triples (l,f,h), we set $\Phi(l,f,h)(t) = 0$ for $t \in [0,T]$.

- Let $g \in \mathbb{C}_0^m$, and define the function $\widetilde{Q}_T$ by

$$\widetilde{Q}_T(g) = \inf_{l,f \in (\mathbb{H}_0^1)^m} \left[\frac{1}{2}\int_0^T ||\dot{l}(s)||_m^2 ds + \frac{1}{2}\int_0^T ||\dot{f}(s)||_m^2 ds : \Phi(l,f,\widehat{f}(t)) = g(t),\, t \in [0,T] \right],$$

  if the equation $\Phi(l,f,\widehat{f}(t)) = g(t)$ is solvable for l and f. If there is no solution, then we set $\widetilde{Q}_T(g) = \infty$.

The Rate Function. Simplifications

Suppose for every $(t,u) \in [0,T] \times \mathbb{R}^d$, the matrix $\sigma(t,u)$ is invertible. Then the following are true:

- For all functions $g \in (\mathbb{H}_0^1)^m$,

$$\widetilde{Q}_T(g) = \frac{1}{2} \inf_{f \in (\mathbb{H}_0^1)^m} \int_0^T (||\bar{C}^{-1}\sigma(s,\widehat{f}(s))^{-1}[\dot{g}(s) - b(s,\widehat{f}(s)) - \sigma(s,\widehat{f}(s))C\dot{f}(s)]||_m^2 + ||\dot{f}(s)||_m^2)ds,$$

  and $\widetilde{Q}_T(g) = \infty$ otherwise.

- The rate function $\widetilde{Q}_T$ is continuous in the topology of the space $(\mathbb{H}_0^1)^m$.

- If $n = 1$, then

$$\widetilde{Q}_T(g) = \frac{1}{2} \inf_{f \in \mathbb{H}_0^1} \int_0^T \left[\frac{(\dot{g}(s) - b(s,\widehat{f}(s)) - \rho\sigma(s,\widehat{f}(s))\dot{f}(s))^2}{(1-\rho^2)\sigma(s,\widehat{f}(s))^2} + \dot{f}(s)^2 \right] ds.$$

Sample Path LDPs for Volatility Processes

We will next formulate sample path LDPs for volatility processes. These LDPs hold under special restrictions on the volatility models (Assumptions (C1) – (C7) that will be explained later).

Theorem 10. *Suppose Assumptions (C1) – (C7) hold, and let $Y^{(\varepsilon)}$ with $Y_0^{(\varepsilon)} = y$ be the solution to the volatility equation in the canonical set-up. Then, the process $Y^{(\varepsilon)}$ satisfies a sample path large deviation principle with speed ε^{-1} and good rate function defined on $\mathcal{W}^d$ by*

$$I_y(\varphi) = \inf_{\{f \in L^2([0,T],\mathbb{R}^m) : \Gamma_y(f) = \varphi\}} \frac{1}{2} \int_0^T ||f(t)||_m^2 dt$$

if $\{f \in L^2([0,T],\mathbb{R}^m) : \Gamma_y(f) = \varphi\} \neq \emptyset$, and $I_y(\varphi) = \infty$ otherwise.

Theorem 11. *Under the restrictions in Theorem 10, the process $\varepsilon \mapsto (\sqrt{\varepsilon}W, \sqrt{\varepsilon}B, \widehat{B}^{\varepsilon})$, $\varepsilon \in (0,1]$, satisfies a sample path large deviation principle with speed ε^{-1} and good rate function defined on $\mathcal{W}^m \times \mathcal{W}^m \times \mathcal{W}^d$ by*

$$\widetilde{I}_y(\varphi_1, \varphi_2, \varphi_3) = \frac{1}{2} \int_0^T ||\dot{\varphi}_1(t)||_m^2 dt + \frac{1}{2} \int_0^T ||\dot{\varphi}_2(t)||_m^2 dt$$

in the case where $\varphi_1, \varphi_2 \in (H_0^1)^m$ and $\varphi_3 = \widehat{\varphi_2}$, and by $\widetilde{I}_y(\varphi_1, \varphi_2, \varphi_3) = \infty$ otherwise.

Controlled Counterparts of Volatility Equations

Let $\mathcal{M}^2[0,T]$ be the space of all $\mathbb{R}^m$-valued square-integrable $\{\mathcal{F}_t^B\}$-predictable processes. The controls will be chosen from the space $\mathcal{M}^2[0,T]$. Deterministic controls will be employed as well. They are functions belonging to the space $L^2([0,T],\mathbb{R}^m)$.

Definition 8. *Let $N > 0$. By $\mathcal{M}_N^2[0,T]$ is denoted the class of controls $v \in \mathcal{M}^2[0,T]$ satisfying the condition $\int_0^T ||v_s||_m^2 ds \leq N$ $\mathbb{P}$-a.s.*

Suppose $v \in \mathcal{M}_N^2[0,T]$. Then, the controlled counterparts of the volatility equations are as follows:

$$Y_t^{(v)} = y + \int_0^t a(t,s,V^{1,v},Y^{(v)})ds + \int_0^t c(t,s,V^{2,v},Y^{(v)})v_s ds + \int_0^t c(t,s,V^{2,v},Y^{(v)})dB_s$$

and

$$V_s^{i,v} = V_0^{(i)} + \int_0^s \bar{b}_i(r,V^{i,v})dr + \int_0^s \bar{\sigma}_i(r,V^{i,v})v_r dr + \int_0^s \bar{\sigma}_i(r,V^{i,v})dB_r, \quad i = 1,2.$$

- Suppose $v \in \mathcal{M}_N^2[0,T]$ for some $N > 0$. It follows from Girsanov's Theorem that the process

$$B_t^{(v)} = B_t + \int_0^t v_s ds, \quad t \in [0,T]$$

is an m-dimensional Brownian motion on $\mathcal{W}^m$ with respect to a measure $\mathbb{P}^{(v)}$ on $\mathcal{F}_T^m$ that is equivalent to the measure $\mathbb{P}$.

- The process $B^{(v)}$ is adapted to the filtration $\{\mathcal{F}_t^B\}$.

- It follows that the controlled volatility equations can be rewritten as follows:

$$Y_t^{(v)} = y + \int_0^t a(t,s,V^{1,v},Y^{(v)})ds + \int_0^t c(t,s,V^{2,v},Y^{(v)})dB_s^{(v)} \tag{3}$$

and

$$V_s^{i,v} = V_0^{(i)} + \int_0^s \bar{b}_i(r,V^{i,v})dr + \int_0^s \bar{\sigma}_i(r,V^{i,v})dB_r^{(v)}, \quad i = 1,2.$$

Assumption (C1)

- For all $(\eta_1, \varphi) \in \mathcal{W}^{k_1} \times \mathcal{W}^d$, the map $(t,s) \mapsto a(t,s,\eta_1,\varphi)$ is Borel measurable, with values in the space $\mathbb{R}^d$.

- For all $(\eta_2, \varphi) \in \mathcal{W}^{k_2} \times \mathcal{W}^d$, the map $(t,s) \mapsto c(t,s,\eta_2,\varphi)$ is Borel measurable, with values in the space of $d \times m$-matrices.

- The maps a and c are of Volterra type in the first two variables.

- For every $t \in [0,T]$, $(s,\eta_1,\varphi) \mapsto a(t,s,\eta_1,\varphi)$ and $(s,\eta_2,\varphi) \mapsto c(t,s,\eta_2,\varphi)$ are predictable path functionals mapping the space $[0,t] \times \mathcal{W}^{k_1} \times \mathcal{W}^d$ into the space $\mathbb{R}^d$ and the space $[0,t] \times \mathcal{W}^{k_2} \times \mathcal{W}^d$ into the space of $d \times m$ matrices, respectively.

- The definition of a predictable path functional can be found in [19] (see Definition (8.3) and Remark (8.4) on p. 122). The requirement above is similar to Convention (8.7) on p. 123 in [19].

Assumption (C2)

(a) Let $\eta_1 \in \mathcal{W}^{k_1}$, $\eta_2 \in \mathcal{W}^{k_2}$, and $\varphi \in \mathcal{W}^d$. Then, the following inequalities hold for all $t \in [0, T]$:

$$\int_0^t ||a(t, s, \eta_1, \varphi)||_d ds < \infty \quad \text{and} \quad \int_0^T ||c(t, s, \eta_2, \varphi)||^2_{d \times m} ds < \infty.$$

(b) For all fixed $\eta_1 \in \mathcal{W}^{k_1}$ and $\varphi \in \mathcal{W}^d$, the function

$$t \mapsto \int_0^t a(t, s, \eta_1, \varphi) ds$$

is a continuous $\mathbb{R}^d$-valued function on $[0, T]$. In addition, for every fixed $t \in [0, T]$ the function

$$(\eta_1, \varphi) \mapsto \int_0^t a(t, s, \eta_1, \varphi) ds$$

is continuous on the space $\mathcal{W}^{k_1} \times \mathcal{W}^d$.

(c) Let $\eta_{2,n} \to \eta_2$ in $\mathcal{W}^{k_2}$ and $\varphi_n \to \varphi$ in $\mathcal{W}^d$ as $n \to \infty$. Then, for every $t \in [0, T]$,

$$\int_0^t ||c(t, s, \eta_{2,n}, \varphi_n) - c(t, s, \eta_2, \varphi)||^2_{d \times m} ds \to 0 \quad \text{as} \quad n \to \infty.$$

Assumption (C3)

(a) For all $0 < \varepsilon \leq 1$ there exists a strong solution to the scaled volatility equation.

(b) Let $v \in M_N^2[0,T]$ for some $N > 0$. Then, any two strong solutions to the equation in (3) are $\mathbb{P}$-indistinguishable.

Remark 12. *Assumption (C3)(b) is weaker than the pathwise uniqueness condition employed in* [5].

Assumption (C4)

For every function $f \in L^2([0,T], \mathbb{R}^m)$, the equation

$$\eta(t) = y + \int_0^t a(t,s,\psi_{1,f},\eta)ds + \int_0^t c(t,s,\psi_{2,f},\eta)f(s)ds, \tag{4}$$

is uniquely solvable in $\mathcal{W}^d$.

Remark 13. *Under the restrictions imposed on $\bar{b}_i$ and $\bar{\sigma}_i$ in [5], the functional equations*

$$\psi_i(s) = V_0^{(i)} + \int_0^s \bar{b}_i(r,\psi_i)dr + \int_0^s \bar{\sigma}_i(r,\psi_i)f(r)dr, \quad i = 1,2,$$

are uniquely solvable, the solutions $\psi_{i,f}$ belong to the spaces $\mathcal{W}^{k_i}$, and if $f_n \mapsto f$ weakly in $L^2([0,T],\mathbb{R}^m)$, then $\psi_{i,f_n} \mapsto \psi_{i,f}$ in $\mathcal{W}^{k_i}$ for $i = 1,2$.

Remark 14. *It is true that the equation in (4) is always solvable. Therefore, only the uniqueness condition must be included in Assumption (C4).*

Definition 15. *The map $\Gamma_y : L^2([0,T],\mathbb{R}^m) \mapsto \mathcal{W}^d$ is defined by $\Gamma_y f = \eta_f$ where η_f is the unique solution to the equation in (4).*

Assumption (C5)

For every $N > 0$, set $D_N = \{f \in L^2([0,T], \mathbb{R}^m) : \int_0^T ||f(t)||_m^2 dt \le N\}$. It is assumed that the restriction of the map Γ_y to D_N is a continuous map from D_N equipped with the weak topology into the space $\mathcal{W}^d$.

<u>*Measurable Functionals:*</u>

Let $v \in M_N^2[0,T]$ for some $N > 0$. Then, there exists a map $g^{(2)} : \mathcal{W}^m \mapsto \mathcal{W}^{k_2}$ satisfying the following conditions:
(i) $g^{(2)}(B) = V^{(2)}$.
(ii) $g^{(2)}(B^{(v)}) = V^{2,v}$ $\mathbb{P}$-a.s.
(iii) For every $t \in [0,T]$, $g^{(2)}$ is $\widetilde{\mathcal{B}}_t^m / \mathcal{B}_t^{k_2}$-measurable.

See Lemma A.1 in [5], see also Theorem 10.4 on p. 126 in [19].

Suppose Assumption (C3) holds. Then there exists a map $h : \mathcal{W}^m \mapsto \mathcal{W}^d$ such that the solution Y to the volatility equation satisfies $Y = h(B)$ and the map h is $\widetilde{\mathcal{B}}_t^m / \mathcal{B}_t^d$-measurable for all $t \in [0,T]$.

Assumption (C6)

The process $t \mapsto \int_0^t c(t,s,g^{(2)}(B^{(v)}),h(B^{(v)}))dB_s^{(v)}$, $t \in [0,T]$ is continuous.

Assumption (C6) looks rather complicated. A special case, where Assumption (C6) is satisfied, is when the map c does not depend on the variable t. Indeed, in such a case, the correctness of Assumption (C6) follows from the restrictions on the map c in Assumption (C2)(a) and the continuity properties of stochastic integrals. More examples of the validity of Assumption (C6) will be provided later.

Assumption (C7)

Suppose $0 < \varepsilon_n < 1$, with $n \geq 1$, is a sequence of numbers such that $\varepsilon_n \to 0$ as $n \to \infty$. Let $v^{(n)}$, $n \geq 1$, be a sequence of controls satisfying the condition $v^{(n)} \in \mathcal{M}_N^2[0, T]$ for some $N > 0$ and all $n \geq 1$ (see Definition 9).

Assumption (C7). (i) The family of $\mathcal{W}^d$-valued random variables $Y^{\varepsilon_n, v^{(n)}}$, with $n \geq 1$, is tight in $\mathcal{W}^d$.

(ii) For every $t \in [0, T]$, the following inequality is satisfied:

$$\sup_{n \geq 1} \int_0^t \mathbb{E}\left[||c(t, s, V^{2, \varepsilon_n, v^{(n)}}, Y^{\varepsilon_n, v^{(n)}})||^2_{d \times m} \right] ds < \infty.$$

First Exit Time

Definition 16. *(i) For every $\varepsilon \in (0,1]$, the first exit time of the scaled log-process from the set O is defined by $\tau^{(\varepsilon)} = \inf\{s \in (0,T] : X_s^{(\varepsilon)} \notin O\}$ if the previous set is not empty, and by $\tau^{(\varepsilon)} = \infty$ otherwise.*
(ii) For every $\varepsilon \in (0,1]$, the first exit time probability function is defined by $v_\varepsilon(t) = \mathbb{P}(\tau^{(\varepsilon)} \leq t)$, $t \in (0,T]$.

In the book [10] of Freidlin and Wentzell, the following restriction on an open set $O \subset \mathbb{R}^m$ was used: There exist interior points of the complement of O arbitrarily close to every point of the boundary of O. The previous condition can be formulated as follows: $\partial O = \partial(\mathrm{ext}(O))$ where $\mathrm{ext}(O)$ is the set of interior points of the complement of O, and, for a set $D \subset \mathbb{R}^m$, the symbol ∂D stands for the boundary of D.

Theorem 17. *Suppose the LDP in Theorem 4 holds. Suppose also that an open set $O \subset \mathbb{R}^m$ satisfies the Freidlin-Wentzell condition. Then*

$$\varepsilon \log \mathbb{P}(\tau^{(\varepsilon)} \leq t) = -\inf_{g \in \mathcal{A}_t} \widetilde{Q}_T(g) + o(1) \quad \text{as } \varepsilon \to 0$$

where $\mathcal{A}_t = \{f \in \mathbb{C}_0^m : f(s) \notin O - x_0 \text{ for some } s \in (0,t]\}$.

Binary Barrier Options

Suppose that the model in (1) describes the dynamics of price processes associated with a portfolio of correlated assets.

We will discuss the small-noise asymptotic behavior of binary up-and-in barrier options.

Denote by $\mathbb{R}^m_+$ the subset of $\mathbb{R}^m$ consisting of all the vectors $s = (s_1, \cdots, s_m) \in \mathbb{R}^m$ such that $s_i > 0$ for all $1 \leq i \leq m$, and let $O \subset \mathbb{R}^m_+$ be an open set. The boundary ∂O of the set O plays the role of the barrier. It is assumed that the model in (1) satisfies the restrictions imposed in Theorem 4.

Definition 18. *Let O be an open set in $\mathbb{R}^m_+$, and suppose that for every $\varepsilon \in (0,1]$ the initial condition s_0 for the process $t \mapsto S_t^{(\varepsilon)}$ satisfies $s_0 \in O$. In a small-noise setting, a binary up-and-in barrier option pays a fixed amount of cash, say one dollar, if the m-dimensional asset price process $S^{(\varepsilon)}$ hits the barrier ∂O at some time during the life of the option.*

The price $B(\varepsilon)$ of the up-and-in barrier option at $t = 0$ is given by

$$B(\varepsilon) = e^{-rT}\mathbb{P}(S_t^{(\varepsilon)} \in \partial O \text{ for some } t \in [0, T])$$

where $r > 0$ is the interest rate.

Binary Barrier Options. Asymptotic Formula

Denote by $\widetilde{O}$ the open subset of $\mathbb{R}^m$ defined by

$$\widetilde{O} = \{x = (x_1, \cdots, x_m) \in \mathbb{R}^m : (e^{x_1}, \cdots, e^{x_m}) \in O\}.$$

Theorem 19. *Suppose the LDP in Theorem 4 holds, and let O sarisfy the Freidlin-Wentzell condition. Then, the following asymptotic formula holds:*

$$\varepsilon \log B(\varepsilon) = - \inf_{g \in \mathcal{A}_T} \widetilde{Q}_T(g) + o(1) \quad as \ \varepsilon \to 0$$

where $\mathcal{A}_T = \{f \in \mathbb{C}_0^m : f(s) \notin \widetilde{O} - x_0 \text{ for some } s \in [0, T]\}$.

Admissible Kernels

The remaining part of the talk is devoted to examples of stochastic volatility models for which Theorem 4 is valid. More precisely, we will explain for what volatility processes $\widehat{B}$ Assumptions (C1)-(C7) hold true.

- Let K be a real function on $[0,T]^2$. We call the function K an admissible Hilbert-Schmidt kernel if the following conditions hold:

  (a) K is Borel measurable on $[0,T]^2$.
  (b) K is Lebesgue square-integrable over $[0,T]^2$.
  (c) For every $t \in (0,T]$, the slice function $s \mapsto K(t,s)$, with $s \in [0,T]$, belongs to the space $L^2[0,T]$.
  (d) For every $t \in (0,T]$, the slice function is not almost everywhere zero.

- If an admissible kernel K satisfies the condition $K(t,s) = 0$ for all $s > t$, then K is called an admissible Volterra kernel.

Volatility Processes in Gaussian Models

- Any admissible Volterra kernel K generates a Hilbert-Schmidt operator

  $$\mathcal{K}(f)(t) = \int_0^t K(t,s)f(s)ds, \quad f \in L^2[0,T], \quad t \in [0,T],$$

  and a Volterra Gaussian process

  $$\widehat{B}_t = \int_0^t K(t,s)dB_s, \quad t \in [0,T].$$

- It is clear that the process $\widehat{B}$ is adapted to the filtration $\{\mathcal{F}_t^B\}_{0\le t\le T}$. This process is used as the volatility process in a one-factor Gaussian stochastic volatility model (see [12]).

- Important examples of such volatility processes are Brownian motion, the Ornstein-Uhlenbeck process, fractional Brownian motion, the Riemann-Liouville fractional Brownian motion, and super rough Volterra type Gaussian processes (see [12, 1]), e.g., logarithmic Brownian motion (see [17]).

- The scaled volatility process is defined as follows: $\widehat{B}_t^{(\varepsilon)} = \sqrt{\varepsilon}\widehat{B}_t$ for $t \in [0,T]$.

Fernique's Condition

- Let $X_t, t \in [0,T]$, be a square integrable stochastic process on $(\Omega, \mathcal{F}, \mathbb{P})$. The canonical pseudo-metric δ associated with this process is defined by the formula

$$\delta^2(t,s) = \mathbb{E}[(X_t - X_s)^2], \quad (t,s) \in [0,T]^2.$$

- Suppose η is a modulus of continuity on $[0,T]$ such that $\delta(t,s) \le \eta(|t-s|)$ for $t,s \in [0,T]$. Suppose also that for some $b > 1$, the following inequality holds:

$$\int_b^\infty \eta\left(u^{-1}\right) (\log u)^{-\frac{1}{2}} \frac{du}{u} < \infty.$$

- The previous condition is called Fernique's condition. It guarantees that the volatility process $\widehat{B}$ is a continuous Gaussian process.

Assumption F and Non-Gaussian Fractional Models

By the Itô isometry, the following equality holds for the process $\widehat{B}$:

$$\delta^2(t,s) = \int_0^T (K(t,u) - K(s,u))^2 du, \quad t,s \in [0,T].$$

The L^2-modulus of continuity of the kernel K is defined on $[0,T]$ by

$$M_K(\tau) = \sup_{t,s\in[0,T]:|t-s|\leq\tau} \int_0^T (K(t,u) - K(s,u))^2 du, \quad \tau \in [0,T].$$

Assumption F. The kernel K is an admissible Volterra kernel such that $M_K(\tau) \leq \eta^2(\tau)$, $\tau \in [0,T]$ for some modulus of continuity η satisfying Fernique's condition.

One-factor non-Gaussian fractional stochastic volatility models:

The volatility process in such a model is given by

$$\hat{B}_t = \int_0^t K(t,s)U(V_s)ds$$

where $U : \mathbb{R} \mapsto [0,\infty)$ is a continuous non-negative function and K is an admissible kernel. The process V is the solution to a diffusion equation satisfying special conditions (see Gerhold, Gerstenecker, A. G. [11]).

Mixed Models

We will next introduce a new class of volatility models. A model belonging to this class may be called a mixture of a multivariate Gaussian stochastic volatility model and a multivariate non-Gaussian fractional model.

The volatility process $Y_t = (Y_t^{(1)}, \cdots Y_t^{(d)})$ in a mixed model satisfies the following system of stochastic differential equations:

$$Y_t^{(i)} = x_i + \int_0^t K_i(t,s) U_i(V_s) ds + \sum_{j=1}^{m} \int_0^t K_{ij}(t,s) dB_s^{(j)}, \quad 1 \le i \le d.$$

<u>*Restrictions:*</u>

(1) K_i, with $0 \le i \le d$, and $\{K_{ij}\}$, with $1 \le i \le d$ and $1 \le j \le m$, are families of admissible Volterra type Hilbert-Schmidt kernels such that Assumption F holds for them.
(2) V is an auxiliary k-dimensional continuous process defined on the space $\mathcal{W}^m$ equipped with the canonical set-up.
(3) Conditions (H1) – (H6) in [5] are satisfied for the process V.
(4) U is a continuous map from $\mathbb{R}^k$ into $\mathbb{R}^d$.

Theorem 20. *Assumptions (C1) – (C7) hold true for the mixed volatility model introduced above. Therefore, the LDPs in Theorems 10 and 11 hold for the mixed model.*

By assuming that $U = 0$, we obtain the scaled volatility process in a multivariate Gaussian stochastic volatility model.

Similarly, the scaled volatility process in a multivariate non-Gaussian fractional model can be obtained by setting $K_{ij} = 0$ for all $1 \leq i \leq d$ and $1 \leq j \leq m$.

The Heston model and the fractional Heston model are special cases of non-Gaussian models described above. In fractional Heston models, the volatility is a fractional integral operator applied to the CIR process.

A different generalization of the Heston model (a rough Heston model) is due to El Euch and Rosenbaum (see [7]). In the rough Heston model, the fractional integral operator is applied to the CIR equation, and not to the CIR process.

I do not know whether the LDP in Theorem 4 holds the log-process in the rough Heston model.

Volterra Type Equations

Multidimensional Volterra type stochastic differential equation:

$$Y_t = y + \int_0^t a(t,s,Y_s)ds + \int_0^t c(t,s,Y_s)dB_s. \tag{6}$$

The scaled version:

$$Y_t^\varepsilon = y + \int_0^t a(t,s,Y_s^\varepsilon)ds + \sqrt{\varepsilon}\int_0^t c(t,s,Y_s^\varepsilon)dB_s, \tag{7}$$

Assumptions Used in the Papers of Wang [22] and Zhang [23]

(H1) For some $p > 2$ there exists $C_T > 0$ such that for all $x, y \in \mathbb{R}^d$ and $s, t \in [0, T]$,

$$||a(t,s,x) - a(t,s,y)||_d \leq C_T K_1(t,s) \rho^{\frac{1}{p}}(||x-y||_d^p),$$
$$||c(t,s,x) - c(t,s,y)||_{d\times m}^2 \leq C_T K_2(t,s) \rho^{\frac{2}{p}}(||x-y||_d^p),$$

and

$$\int_0^t (||a(t,s,0)||_d + ||c(t,s,0)||_{d\times m}^2) ds \leq C_T$$

where K_i, with $i = 1, 2$, are two positive functions on $[0, T]^2$ for which

$$\int_0^t \left[K_1(t,s)^{\frac{p}{p-1}} + K_2(t,s)^{\frac{p}{p-2}} \right] ds \leq C_T, \quad t \in [0, T].$$

In addition, $\rho : \mathbb{R}^+ \mapsto \mathbb{R}^+$ is a concave function satisfying $\int_{0+}^{\cdot} \rho(u)^{-1} du = \infty$.

(H2) For all $t, t', s \in [0, T]$ and $x \in \mathbb{R}^d$,

$$||a(t,s,x) - a(t',s,x)||_d \leq F_1(t',t,s)(1 + ||x||_d),$$

$$||c(t,s,x) - c(t',s,x)||_{d\times m}^2 \leq F_2(t',t,s)(1 + ||x||_d^2),$$

and for some $C > 0$ and $\theta > 1$,

$$\int_0^t (||a(t,s,0)||_d^\theta + ||c(t,s,0)||_{d\times m}^{2\theta})ds < C.$$

The functions F_i, $i = 1, 2$, are positive functions on $[0, T]^3$ satisfying the condition

$$\int_0^{t\wedge t'} (F_1(t',t,s) + F_2(t',t,s))ds \leq C|t - t'|^\gamma$$

for some $\gamma > 0$.

Results obtained in Wang [22]:

- If Condition (H1) holds, then there exists a unique progressively measurable solution Y to the equation in (6).

- Moreover, if Conditions (H1) and (H2) hold, then the unique solution Y has a δ-Hölder continuous version for any $\delta \in (0, \frac{1}{p} \wedge \frac{\theta-1}{2\theta} \wedge \frac{\gamma}{2})$.

A slightly weaker condition than Condition (H2):

- $(\widehat{H}2)$ The restriction $t, t', s \in [0, T]$ in Condition (H2) is replaced by the restriction $0 \leq s \leq t, t' \leq T$.

It is not hard to see, by analyzing the main results obtained in the paper [22] of Wang, that these results hold true with Condition (H2) replaced by Condition $(\widehat{H}2)$.

Results obtained in Zhang [23]:

- Zhang established a sample path LDP for the unique solution $\varepsilon \mapsto Y^{(\varepsilon)}_{\cdot}(\cdot)$, $\varepsilon \in (0,1]$ to the equation in (7), under Conditions (H1) and (H2) and two extra conditions (H3) and (H4) (see Theorem 1.2 in [23]).

- Note that the initial condition $y \in \mathbb{R}^d$ plays the role of a variable in the process defined above. The state space of this process is the space of continuous maps from $[0,T] \times \mathbb{R}^d$ into $\mathbb{R}^d$.

- Using the LDP obtained by Zhang and the contraction principle, we prove a sample path LDP for the process $\varepsilon \mapsto Y^{(\varepsilon)}$, with the initial condition $y \in \mathbb{R}^d$ that is fixed. The state space of this process is the space $\mathcal{W}^d$.

The LDP obtained by Zhang is a special case of the universal LDP:

Theorem 21. *Suppose Conditions (H1) and $(\widehat{H}2)$ hold true for the maps a and c appearing in (6). Then, Assumptions (C1) - (C7) are satisfied. Therefore, under Conditions (H1) and $(\widehat{H}2)$, the LDPs in Theorems 10 and 11 hold for the process $\varepsilon \mapsto Y^{(\varepsilon)}$.*

Remark: Conditions (H3) and (H4) are not needed in the LDP for the process $\varepsilon \mapsto Y^{(\varepsilon)}$.

Assumptions in Nualart - Rovira [18].

(H_1) The map a is measurable from $\{0 \leq s \leq t \leq T\} \times \mathbb{R}^d$ to $\mathbb{R}^d$, while the map c is measurable from $\{0 \leq s \leq t \leq T\} \times \mathbb{R}^d$ to $\mathbb{R}^{d\times m}$.
(H_2) The maps a and c are Lipschitz in x uniformly in the other variables, that is,

$$||c(t,s,x) - c(t,s,y)||_{d\times m} + ||a(t,s,x) - a(t,s,y)||_d \leq K||x-y||_d$$

for some constant $K > 0$, all $x, y \in \mathbb{R}^d$, and all $0 \leq s \leq t \leq T$.
(H_3) The maps a and c are α-Hölder continuous in t on $[s,T]$ uniformly in the other variables. This means that there exists a constant $K > 0$ such that

$$||c(t,s,x) - c(r,s,x)||_{d\times m} + ||a(t,s,x) - a(r,s,x)||_d \leq K|t-r|^\alpha$$

for all $x \in \mathbb{R}^d$ and $s \leq t, r \leq T$ where $0 < \alpha \leq 1$.
(H_4) There exists a constant $K > 0$ such that

$$||c(t,s,x) - c(r,s,x) - c(t,s,y) + c(r,s,y)||_{d\times m} \leq K|t-r|^\gamma||x-y||_d$$

for all $x, y \in \mathbb{R}^d$ and $T \geq t, r \geq s$ where $0 < \gamma \leq 1$.
(H_5) $a(t,s,x_0)$ and $c_j(t,s,x_0)$ are bounded.

In [18], a sample path LDP was established for the unique solution to the equation in (7) under Conditions $(H_1) - (H_5)$ (see Theorem 1 in [18]).

<u>*Our result:*</u>

Theorem 22. *Conditions $(H_1) - (H_3)$ and (H_5) in* [18] *imply Conditions (H1) and $(\widehat{H}2)$ in* [22, 23]. *Therefore, the LDPs in Theorems 10 and 11 hold for the process $\varepsilon \mapsto Y^{(\varepsilon)}$ in the canonical set-up (see Theorem 21).*

Remark: Theorem 1 in [18] is valid under Conditions (H_1) - (H_3) and (H_5) if the canonical set-up is employed. Condition (H_4) is not needed. We do not know if the same is true on any set-up.

References I

[1] C. Bayer, F. Harang, P. Pigato, Log-modulated rough stochastic volatility models, SIAM J. Financial Math. 12 (2021) 1257-1284.

[2] C. Bayer, P. K. Friz, P. Gassiat, J. Martin, B. Stemper, A regularity structure for rough volatility, Math. Finance 30 (2020) 782-832.

[3] G.Catalini, B. Pacchiarotti, Asymptotics for multifactor Volterra type stochastic volatility models, available on arXiv:2109.09448.

[4] M. Cellupica, B. Pacchiarotti, Pathwise asymptotics for Volterra type stochastic volatility models, J. Theor. Probab. 34 (2021) 682-727.

[5] A. Chiarini, M. Fischer, On large deviations for small noise Itô processes, Adv. in Appl. Probab. 46 (2014) 1126-1147.

[6] P. Dupuis, R. S. Ellis, A Weak Convergence Approach to the Theory of Large Deviations, John Wiley & Sons, Inc., 1997.

[7] O. El Euch, M. Rosenbaum, The characteristic function of rough Heston models, Math. Finance 29 (2019) 3-38.

[8] X. Fernique, Continuité des processus Gaussiens, C. R. Acad. Sci. Paris 258 (1964) 6058-6060.

[9] M. Forde, H. Zhang, Asymptotics for rough stochastic volatility models, SIAM J. Financial Math. 8 (2017) 114-145.

[10] M. I. Freidlin, A. D. Wentzell, Random Perturbations of Dynamical Systems, Springer-Verlag New York, 1998.

[11] S. Gerhold, C. Gerstenecker, A. Gulisashvili, Large deviations for fractional volatility models with non-Gaussian volatility driver, Stochastic Process. Appl. 142 (2021) 580-600.

References II

[12] A. Gulisashvili, Time-inhomogeneous Gaussian stochastic volatility models: Large deviations and super-roughness, Stochastic Process. Appl. 139 (2021) 37-79.

[13] A. Gulisashvili, Large deviation principles for stochastic volatility models with reflection and three faces of the Stein and Stein model, submitted for publication, available on arXiv:2006.15431, 2020.

[14] A. Gulisashvili, Gaussian stochastic volatility models: Scaling regimes, large deviations, and moment explosions, Stochastic Process. Appl. 130 (2020) 3648-3686.

[15] A. Gulisashvili, Large deviation principle for Volterra type fractional stochastic volatility models, SIAM J. Financial Math. 9 (2018) 1102-1136.

[16] A. Jacquier, A. Pannier, Large and moderate deviations for stochastic Volterra systems, available on arXiv:2004.10571, 2020.

[17] O. Mocioalca, F. Viens, Skorokhod integration and stochastic calculus beyond the fractional Brownian scale, J. Funct. Anal. 222 (2005) 385-434.

[18] D. Nualart, C. Rovira, Large deviations for stochastic Volterra equations, Bernoulli 6 (2000) 339-355.

[19] L. C. G. Rogers, D. Williams, Diffusions, Markov Processes, and Matringales, Vol. 2, Itô Calculus, Cambridge University Press, 2000.

[20] C. Rovira, M. Sanz-Solé, Large deviations for stochastic Volterra equations in the plane, Potential Anal. 12 (2000) 359-383.

[21] S. R. S. Varadhan, Asymptotic probabilities and differential equations, Comm. Pure Appl. Math. 19 (1966) 261-286.

[22] Z. Wang, Existence and uniqueness of solutions to stochastic Volterra equations with singular kernels and non-Lipschitz coefficients, Statist. Probab. Lett. 78 (2008) 1062-1071.

[23] X. Zhang, Euler schemes and large deviations for stochastic Volterra equations with singular kernels, J. Differental Equations 244 (2008) 2226-2250.

Thank you!