Dynamics and integrability analysis of the swinging Atwood generalisations

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Abstract: We consider dynamics and integrability of certain generalizations of 3-dimensional swinging Atwood's machine systems with additional Coulomb's and Hooke's interactions. We present the complexity of these models by means of numerous methods and techniques, among others: the Poincare cross-sections, the bifurcations diagrams, the Lyapunov exponents spectra, etc. We show that in general the presented models are not integrable and they manifest highly chaotic dynamics. However, for specific values of parameters they admit additional first integrals. For integrable cases we find additional first integrals and we construct general solutions written in terms of Jacobi and Weierstrass elliptic functions which is quite peculiar for Hamiltonian systems with more than one degree of freedom. Moreover, we present bifurcation diagrams for integrable cases and we find resonance curves, which give families of periodic orbits of the systems. Finally, we show that under the absence of the gravity, both models are even super-integrable, possessing the Bertrand property.

Keywords: Chaotic modeling, Nonlinear dynamics, Hamiltonian systems, Chaos, Integrability.