Universal bounds in Lorenz-like systems

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The well-known Lorenz-63 system:

$$\begin{aligned} \dot{x} &= \sigma(y - x) \\ \dot{y} &= rx - y - xz \\ \dot{z} &= -bz + xy. \end{aligned} \tag{1}$$

describes the interaction of three modes in the problem of a convection in a two-dimensional fluid. The expression xy has a physical meaning, namely, it is the transport of heat energy, and the knowledge of the properties of this value is important for the understanding the properties of the convection.

System (1) possesses a Lyapunov function, so that all the solutions come inside a certain ball and stay there forever. This means, that for the time averages of physical values on large time intervals will be well-defined and bounded. In [1] the upper bound of the heat transport in the Lorenz system was computed. This value is achieved in the equilibrium point.

In the current work I consider system (1) with added constant offsets, these systems were introduced in [2], [3], moreover, in [2] the upper bound was computed, but it is not achieved on any solution. The results of the present work are the improvement of the heat transport upper bound obtained in [2] to a sharp value (achieved in the equilibrium) and the computation of a sharp upper bound in system from [3].

References

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