Self-Avoiding Models of Moving Polymers and Random Surfaces

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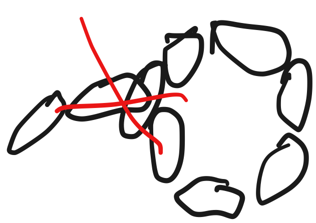
PR5E

① Polymers



Model: Let (S_n) be a nearest-neighbor simple RW on $\mathbb{Z}^d$

All self-avoiding paths of length n have equal prob. $1/Z_n$

$$Q_n(\omega^{(n)}) = \frac{1}{Z_n}$$

Ansatz: $E_n\left[|S_n|^2\right] \approx n^{2\nu}$

d	ν	proved	extend?
1	1	Bolthausen	yes
		Greven - den Hollander	no
2	$\frac{3}{4}$	no, $SLE_{8/3}$	no
3	.588...	no	no
4	$\frac{1}{2}$	no	no
≥ 5	$\frac{1}{2}$	Hara-Slade lace expansion	no

Weakly self-avoiding

$$\ell_n(y) = \#\{k \leq n : S_k = y\}$$

$$Q_n(\omega) = \frac{\exp\left(-\beta \sum_y \ell_n^2(y)\right) P_n}{Z_n}$$

$\mathcal{E}_n = \exp(\ )$ in numerator

Moving polymers

$u(t, x)$ $\quad x$ = length along polymer

$x \in [0, J]$, $u \in \mathbb{R}^1$

$$\partial_t u = \partial_x^2 u + \dot{W}(t, x)$$

Neumann boundary data

$$L_t(A) = m\{x \in [0, J] : u(t, x) \in A\}$$

$$\ell_t(y) = \frac{d L_t(y)}{dy}$$

$$\mathcal{E}_T = \exp\left(-\beta \int_0^T \int_{\mathbb{R}} \ell_t(y)^2 \, dy \, dt\right)$$

$$\frac{dQ_T}{dP_T} = \frac{\mathcal{E}_T}{Z_T} \qquad Z_T = E^{P_T}[\mathcal{E}_T]$$

Result: $\bar{u}(t) = \frac{1}{J}\int_0^J u(t,x)\,dx$

$$R(T,J) = \left[\frac{1}{TJ}\int_0^T \int_0^J (u(t,x) - \bar{u}(t))^2 \, dx \, dt\right]^{\frac{1}{2}}$$

Thm under Q_T, as $T \to \infty$

$$R(T,J) \approx \beta^{\frac{1}{3}} J^{\frac{5}{3}}$$

Conjecture: $R \approx J^{\frac{5}{4}}$

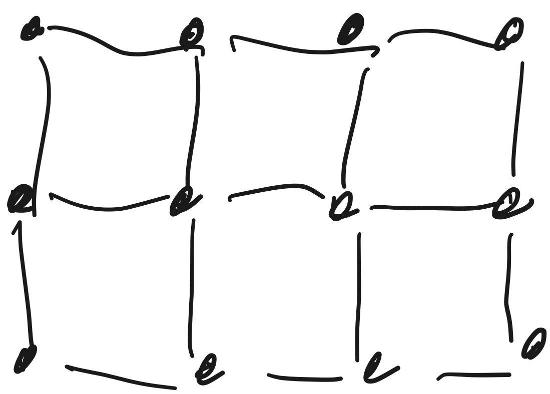
$u \in \mathbb{R}^2$

Random surfaces

Kantor, Kardar, Nelson 86, 87

↓

KPZ



$$P_N = \frac{1}{Z_N} \exp\left(-\beta \sum_{x \sim y} |u(x) - u(y)|^2\right)$$

$$x, y \in \left([-N, N] \cap \mathbb{Z}\right)^d$$

$$u \in \mathbb{R}^D$$

Require $\sum_x u(x) = 0$

$$\mathcal{E}_N = \exp\left(-\gamma \sum_{x,y} 1_{B_1(0)}\left(|u(x) - u(y)|\right)\right)$$

$$\frac{dQ_N}{dP_N} = \frac{1}{Z_N} \mathcal{E}_N$$

$$R_N = \sup_{x,y} | u(x) - u(y) |$$

<u>Results</u> As $N \to \infty$, with $\exists\, \varepsilon_0, K_0 > 0$

Q_N. approach 1

$d = D = 2$

$$\varepsilon_0 \left(\frac{\gamma}{\gamma+\beta}\right)^{\frac{1}{2}} N (\log N)^{-\frac{1}{2}} \lesssim R_N$$

$$\lesssim K_0 \left(\frac{\beta+\gamma}{\beta}\right)^{\frac{1}{2}} N (\log N)^{\frac{3}{2}}$$

<u>$d = D \geq 3$</u>

$$N \lesssim R_N \lesssim N^{\frac{d}{2}}$$

More recently

$d = 2$; $D = 1$

$$R_N \simeq N^{\frac{4}{3}}$$

$d \geq 3$, $1 \lesssim D \lesssim d$

$$N^{\frac{1}{D}\left(d - \frac{2(d-D)}{D+2}\right)} \lesssim R_N$$

$$\lesssim N^{\frac{d}{2} + \frac{d-D}{D+2}}$$

Proof ideas (for 2-d (self-weakly avoiding walk))

$$dQ_N = \frac{dP_N \, \mathcal{E}_N}{Z_N}$$

Upper bound on Q_N

$$Q_N\left(cN^{\frac{3}{4}} \lesssim R_N \lesssim CN^{\frac{3}{4}}\right) \to 1$$

Upper bound on numerator

Lower bound on Z_n

- Use Jensen
- Make a change of prob, so RW, is ballistic

<u>Numerator</u>

$$A_< = \left\{ \sup_{0 \le i \le N} |S_i| \lesssim R \right\}$$

$$A_> = \left\{ \quad \ldots \quad \gtrsim \ldots \right\}$$

Want $Q_N(A_<, A_>)$ small

On $A_<$, $\mathcal{E}_N$ is maximized if S_i is evenly spread over $[-R, R]^2$, ie $l_N(y) = \frac{N}{R^2}$

$$\mathcal{E}_N \approx \exp\left(-\beta \sum_z l_N^2(z)\right)$$

$$= \exp\left(-\beta R^2 \left(\frac{N}{R^2}\right)^2\right)$$

$$= \exp\left(-\beta \frac{N^2}{R^2}\right)$$

$$Q_N(A_>) = E^{P_N}\left[\mathcal{E}_N 1_{A_>}\right]$$

$$\lesssim E^{P_N}\left[1_{A_>}\right]$$

$$\approx \exp\left(-\frac{R^2}{N}\right)$$

Set exponents equal to optimize

$$\frac{R^2}{N} = \frac{N^2}{R^2}$$

$$R \approx N^{\frac{3}{4}}$$ (Flory theory)