Chaotic coorbital motions to L_3 in the Restricted Planar Circular 3-Body Problem

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Abstract

The Restricted 3-Body Problem models the motion of a body of negligible mass under the gravitational influence of two massive bodies, called the primaries. If the primaries perform circular motions and the massless body is coplanar with them, one has the Restricted Planar Circular 3-Body Problem (RPC3BP). In synodic coordinates, it is a two degrees of freedom Hamiltonian system with five critical points, $L_1, .., L_5$, called the Lagrange points.

The Lagrange point L_3 is a saddle-center critical point (collinear with the primaries and beyond the largest one) with a 1-dimensional stable and unstable manifold. When the ratio between the masses of the primaries μ is small, the modulus of the hyperbolic eigenvalues are weaker, by a factor of order $\sqrt{\mu}$, than the elliptic ones.

In this work, we present an asymptotic formula for the distance between the stable and unstable manifolds of L_3. Due to the rapidly rotating dynamics, this distance is exponentially small with respect to $\sqrt{\mu}$ and, as a result, classical perturbative methods (i.e the Melnikov-Poincaré method) can not be applied.

By means of this result, we prove that the stable and unstable invariant manifolds of Lyapunov periodic orbits exponentially close to L_3 intersect transversally. By the Smale-Birkhoff homoclinic theorem, this implies the existence of chaotic motions exponentially close to L_3 and its invariant manifolds.

One of the main challenges of this proof is to approximate the RPC3BP by an averaged integrable Hamiltonian system which possesses a saddle center with a homoclinic orbit and to analyze the complex singularities of its time parameterization. To obtain the leading term of the difference, the perturbed manifolds have to be analtyically extended and analyzed close to these singularities. Lastly, verifying that the invariant manifolds of the Lyapunov periodic orbits behave regularly with respect to those of L_3, we are able to prove that they intersect transversally.

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