Analytical investigation of a galactic refraction billiard

Irene De Blasi

University of Turin
irene.deblasi@unito.it

A new type of dynamical model, describing the motion of a point-mass particle in an elliptic galaxy with a massive central core (such as, for example, a Black Hole), is studied. This kind of model belongs to the more general class of the refraction billiards, which reveal themselves to be particularly useful as a way to describe the motion of a particle subjected to the action of a discontinuous potential. Indeed, in our case the boundary of the billiard does not represent a rigid barrier for the motion of the particle, but instead a refraction interface through which a refraction Snell's law must be satisfied. In our case, the refraction interface is represented by the boundary of a regular and bounded domain of the plane; a Keplerian potential with positive energy acts in the domain, while, outside of it, a two-dimensional homogeneous harmonic potential rules the motion of the particle. The dynamical properties of the system depend crucially on the geometric features of the domain's boundary, and can be studied from different points of view and by means of various techniques. A first study can be performed, for example, in order to analyze the linear Lyapunov stability properties of the equilibrium trajectories, focusing one's attention on the possible bifurcation phenomena. After constructing a suitable first return map, as customary in billiards theory, one can then consider the problem of studying the rotation numbers of the corresponding orbits for different initial conditions. In this framework, Aubry-Mather and KAM theories provide crucial tools to prove the existence of orbits with given rotation numbers for nearly-integrable refraction billiards. The last problem taken into consideration, in continuity with the classical Birkhoff case, is the possible onset of chaotic behaviours, which can be investigated both numerically and, in a more rigorous way, searching for a symbolic dynamics associated to the model.

References

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