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The Regularity problem for divergence form elliptic equations: the Carleson condition

Abstract: Let Ω be an unbounded domain above a Lipschitz graph and $A : \Omega \to M_n(\mathbb{R})$ be a bounded matrix valued function such that

$$A_{ij}(X)\xi_i\xi_j \geq \lambda_0|\xi|^2 \qquad \text{(ellipticity)}$$

for some $\lambda_0 > 0$, all $\xi \in \mathbb{R}^n$ and a.e. $X \in \Omega$. This class of operators has been widely investigated since the Dirichlet problem with data in some L^p space was shown to be solvable for p sufficiently large ([KP]). In particular, it is known that the Dirichlet problem ([DPP]) as well as the Regularity and Neumann problems ([DPR]) are solvable in the full range $1 < p < \infty$ when the Carleson condition is satisfied with a sufficiently small constant. In joint work with M. Dindoš and S. Hofmannn, we can obtain solvability of the Regularity problem in all dimensions without assuming smallness of the Carleson. We have just recently learned that M. Mourgoglu, B. Poggi, and J. Tolsa have proven the same result, by different methods, in domains where the geometry is much less constrained (uniformly rectifiable).