Computation of the asymptotic wavenumber of spiral waves of the Ginzburg Landau equation

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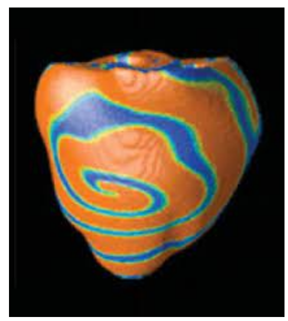
Spiral patterns are commonly observed in certain chemical, biological and physical systems as

Belousov-Zhabotinskii reaction

Social amoebas Dictyostelium discoideium



Cardiac muscle tissue

These systems are governed by a chemical or biological reaction and a spatial diffusion, and they are called reaction-diffusion systems:

$$\partial_\tau U = D\Delta U + F(U, a).$$

Here D is the diffusion matrix, F is the reaction nonlinearity, a is a parameter (for instance some catalyst concentration) and the solution $U = U(\tau, x, y) \in \mathbb{R}^2$.

Our work focuses on the existence of rotating archimedian spirals for the Ginzburg-Landau systems which corresponds to the first order of a reaction-diffusion equation near a Hopf bifurcation:

$$\begin{aligned} u_t &= \Delta u + \lambda\big(\sqrt{u^2+v^2}\big)u - \omega\big(\sqrt{u^2+v^2}\big)w, \\ w_t &= \Delta w + \omega\big(\sqrt{u^2+v^2}\big)u + \lambda\big(\sqrt{u^2+v^2}\big)w, \end{aligned}$$

where the functions λ, ω are $\lambda(z) = 1 - z^2$, $\omega(z) = \omega_0 + qz^2$ and q the small twist parameter. Another rellevant parameter is the asymptotic wavenumber k defined by $q(1-k^2) = \Omega - \omega_0$. It can be seen that the rotating archimedian spirals are solutions of the form $U(t, r, \theta) = f(r)e^{i\,[\Omega t + n\theta - \chi(r)]}$ where $f, f', v = \chi'$ are solutions of a first order system of ordinary differential equations depending on q and k, satisfying $f(r) > 0$, $v(r) < 0$ and the boundary conditions:

$$f(0) = v(0) = 0, \qquad \lim_{r\to\infty} f(r) = \sqrt{1-k^2}, \qquad \lim_{r\to\infty} v(r) = -k.$$

That is, four boundary conditions in a three dimensional system of ordinary differential equations have to be imposed and this suggest a selection mechanism for k with respect to q. Indeed we have proven that, in order to exist rotating archimedian spirals for the Ginzburg-Landau systems, the asymptotic wavenumber has to be

$$k = k(q) = \frac{2}{q} A_n e^{-\frac{\pi}{2nq}} (1 + \mathcal{O}(q))$$

with A_n a constant that only depends on the solutions for $q = 0$.

These systems have been previously studied by many authors, Koppel, Hagan, Greenberg, Chapman, etc. using different techniques as Fenichel's theory, asymptotic methods, numerical methods, shooting methods. Our study is based on functional and complex analysis techniques.

This is a joint work with M. Aguareles (U. de Girona) and T.M. Seara (U. Politècnica de Catalunya).