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Title: Dynamics near the homoclinic set of a slow-fast Hamiltonian system

In slow-fast systems, fast variables change with the rate of order one, and slow variables with the rate of order $\epsilon\ll 1$. The system obtained for $\epsilon=0$ is called frozen. If the frozen Hamiltonian system has one DOF, then in the region where the level curves of the frozen Hamiltonian are closed, there is an adiabatic invariant. A.\,Neishtadt showed that when the fast variable crosses a separatrix of the frozen system, the adiabatic invariant has quasirandom jumps of order ϵ. We partially extend Neishtadt's result to slow-fast Hamiltonian systems with many DOF. If the frozen system has a hyperbolic equilibrium possessing transverse homoclinics, for small ϵ there are local analogs of adiabatic invariants for trajectories in a neighborhood of the homoclinic set. The slow variables evolve in a quasirandom way, shadowing trajectories of systems whose Hamiltonians are these adiabatic invariants. This extends the work of V.\,Gelfreich and D.\,Turaev who considered similar phenomena away from critical points of the frozen Hamiltonian.