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Title: Semi-algebraic geometry and generic long-time stability of nearly-integrable Hamiltonian systems (joint work with Santiago Barbieri)

Abstract: A well-known theorem due to Nekhoroshev in 1977 shows that if we consider a sufficiently regular integrable Hamiltonian which satisfy a transversality property - known as "steepness" – then, adding a smooth small enough Hamiltonian perturbation, the solutions of the perturbed system exist and are stable over a very long time. Nekhoroshev also showed in [1] that the steepness is an open property, which is generic both in measure and topological sense in the space of jets (Taylor polynomials) of sufficiently smooth functions. Surprisingly, this latter result remained almost unstudied up to now, while the other parts of Nekhoroshev's theory received a lot of attention over the decades. Moreover, the definition of steepness is not constructive and no general criteria to ensure that a given function is steep existed up to now. This is a major problem in view of the applications.

We revisit Nekhoroshev's article [1] about the genericity of steepness and connect this proof with much more recent seminal articles of Roytwarf-Yomdin [2] and Yomdin [3] in semi-algebraic geometry. This allows to give a good setting for the considered problem and to clarify the reasoning.

In this framework, thanks to a deep understanding of the genericity of steepness, Santiago Barbieri has developed an explicit algebraic criterion in the space of jets of any order and any number of variables which ensure that a given function is steep.

Reference:

[1] N. N. Nekhoroshev: Stable lower estimates for smooth mappings and for gradients of smooth function, *Mathematics of the USSR-Sbornik*, 1973, vol. 90 (132), no. 3, pp.432-478.

[2] N. Roytwarf and Y. Yomdin: Bernstein Classes, *Annales de l'Institut Fourier*, 47(3):825–858, 1998.

[3] Y. Yomdin: Analytic reparametrization of semi-algebraic sets, *J. Complexity*, 24:54–76, 2008.