Stability estimates for geocentric satellites: Normal forms and Nekhoroshev theorem

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Providing estimates for the long-term stability of a small body moving around the Earth is of crucial importance from both a theoretical and a practical point of view. This goal can be achieved by applying the analytical techniques proper of the Normal form theory, through which, within a Hamiltonian setting, one can study the stability of the body's orbital parameters. In particular, in the framework of the J_2 model (where only the Earth's gravitational influence is considered, taking into account its oblate shape), a normal form construction can be performed to obtain stability estimates for the body's semimajor axis. On the other hand, if one considers the *geolunisolar* model (obtained by taking into consideration also the attractions of Sun and Moon as well), a normalization algorithm allows us to estimate the stability of its eccentricity and inclination simultaneously, in the sense that the stability of a particular quantity depending on both these orbital parameters, the Lidov-Kozai integral, can be studied.
Normal form constructions are also used in the proof and applications of the Nekhoroshev theorem, a breakthrough result through which one can obtain stability estimates for quasi-integrable systems for exponentially long times. Among the hypotheses of the Nekhoroshev theorem we can find the non-degeneracy *steepness* condition, which is implied by stronger hypotheses such as convexity, quasi-convexity or "three-jet" non-degeneracy. Concerning the geocentric satellite's motion, it results that, while the J_2 model is three-jet non-degenerate, the geolunisolar model is quasi-convex: since the three-jet non degeneracy is a weaker condition than the quasi-convexity, this implies that adding the lunisolar gravitational potential to the J_2 one lowers the degeneracy of our model. An alternative version of the Nekhoroshev theorem, based on the absence of resonances rather than non-degeneracy conditions, can be considered to obtain exponential stability estimates. This result has been applied, after a suitable normalization procedure, to the geolunisolar model for different values of the orbital parameters. The results show that the domain of applicability of this version of the theorem (namely, the values of the orbital parameters for which the hypotheses of the theorem are satisfied) is physically relevant up to distances, in terms of semimajor axis, of about 20000 km. In particular, up to 11000 km, stability times of the order of thousands of years have been obtained for a large set of inclinations and eccentricities.

References

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