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Semi-algebraic geometry and effective hamiltonian stability

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Nekhoroshev's theory shows that the solutions of a sufficiently regular integrable hamltonian system - verifying a transversality property on its gradient known as "steepness" - are stable over a long time under the effect of a suitably small perturbation. Nekhoroshev also showed (1973) that the steepness property is generic, both in measure and topologic sense, in the space of jets (Taylor polynomials) of sufficiently smooth functions. However, the proof of this result kept being poorly understood up to now and, surprisingly, the paper in which it is contained is hardly known, whereas the rest of the theory has been widely studied over the decades. Moreover, the definition of steepness is not constructive and no general rule to establish whether a given function is steep or not existed up to now, thus entailing a major problem in applications.

This poster will contain

1. An explanation of the main ideas behind Nekhoroshev's proof of the genericity of steepness by making use of a more modern language. Indeed, the proof strongly relies on arguments complex analysis and semi-algebraic geometry: the latter was much less developed than nowadays at the time that Nekhoroshev was writing, so that many passages appear to be quite obscure in the original article.

2. The description of an important result of real algebraic geometry (a kind of Bernstein-Remez inequality for algebraic functions) that was buried in the proof and seems to have been rediscovered and generalized in many ways by Roytwarf and Yomdin more than twenty-five years after Nekhoroshev's original article.

3. New explicit general algebraic criteria to establish whether a given function is steep or not. Indeed, a deep understanding of the proof of the genericity of steepness allows to determine the algebraic properties satisfied by the jets of steep functions.

Reference: N. N. Nekhoroshev, "Stable lower estimates for smooth mappings and for gradients of smooth function", Mathematics of the USSR-Sbornik, 1973, vol. 90 (132), no. 3, pp.432-478.