Splitting of separatrices for rapidly forced pendulum with a perturbation without first harmonic

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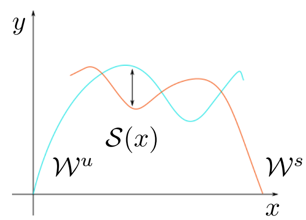
When an equilibrium point in a planar integrable Hamiltonian system has coincident stable and unstable invariant manifolds we refer to this homoclinic trajectory as separatrix. Applying a perturbation can lead to its splitting and the intersection between the manifolds. One way of measuring this splitting is to find an asymptotic expression for the distance between stable and unstable manifolds. In the case of periodic perturbation one usually looks at the distance, $\mathcal{S}(x)$, between the stable ($\mathcal{W}^s$) and unstable ($\mathcal{W}^u$) manifolds of the associated stroboscopic map, as represented below:



In this work we study the splitting of separatrices of a periodically rapidly perturbed planar Hamiltonian system where the perturbation does not have a first harmonic. In particular, we study the classical rapidly perturbed pendulum $H(x,y,t) = \frac{1}{2}y^2 + (\cos(x) - 1) + \mu(\cos(x) - 1)g\left(\frac{t}{\varepsilon}\right)$ with $g(\tau) = \sum_{|k|>1} g^{[k]}e^{ik\tau}$ a 2π-periodic function and $\mu, \varepsilon \ll 1$ with the goal of determining the order of the splitting and providing an asymptotic formula. Systems of this kind undergo exponentially small splitting and have been widely studied. In a perturbative setting, i.e. $\mu \ll 1$, it is known that the Melnikov function actually gives an asymptotic expression for the splitting function provided $g^{[\pm 1]} \neq 0$.

Our study aims at understanding the splitting in the perturbative case where $g^{[\pm 1]} = 0$ and it is motivated by two main reasons. On the one hand the general understanding of the splitting, as current results fail for a perturbation as simple as $g(\tau) = \cos(5\tau) + \cos(4\tau) + \cos(3\tau)$. On the other hand, in a more applied context, a study of the splitting of invariant manifolds of tori of rational frequency p/q in Arnold's original model for diffusion leads to the consideration of Hamiltonians of the form $H(x,y,t) = \frac{1}{2}y^2 + (\cos(x) - 1) + \mu(\cos(x) - 1)\left(\sin\left(p \cdot \frac{t}{\varepsilon}\right) + \cos\left(q \cdot \frac{t}{\varepsilon}\right)\right)$, where, for most $p, q \in \mathbb{Z}$ the perturbation satisfies $g^{[\pm 1]} \neq 0$.

To tackle the question we use a splitting formula based on the solutions of the inner equation and make use of the Hamilton-Jacobi formalism to extract information about the different powers in μ of the splitting. In our main result we show that the leading exponentially small term appears at order μ^n, where n is an integer determined exclusively by the harmonics of the perturbation. As was expected, the Melnikov function is in fact not a correct approximation for the splitting in this case.

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