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The weak-type Carleson theorem

Abstract:

This talk is based on joint work with **Anastasios Fragkos** (Washington U St. Louis). We prove that the weak-$L^{1+\varepsilon}$ norms, and in fact the sparse $(1+\varepsilon, 1)$-norms, of the Carleson maximal partial Fourier sum operator are $\lesssim \varepsilon^{-1}$ as $\varepsilon \to 0^+$. This is an improvement on the restricted weak type estimate of the Carleson-Hunt theorem. Furthermore, our sparse bounds imply new results at the endpoint $p = 1$. In particular, we obtain that the Fourier series of functions from the weighted Arias de Reyna space $\mathrm{QA}_\infty(w)$, which contains the weighted Antonov space $L \log L \log\log\log L(\mathbb{T}; w)$, converge almost everywhere whenever $w \in A_1$. This is an extension of the results of Antonov and Arias De Reyna, where w must be Lebesgue measure.

The main step of the proof is a sharply quantified near-L^1 Carleson embedding theorem for the modulation-invariant wave packet transform. In turn, the latter result exploits a newly developed smooth multi-frequency decomposition which, near the endpoint $p = 1$, outperforms the abstract Hilbert space approach of past works, including the seminal one by Nazarov, Oberlin and Thiele. As a further example of application, we obtain a quantified version of the family of sparse bounds for the bilinear Hilbert transforms due to Culiuc, Ou and myself.