Title: Singular Integral Operators on the Fock Space

Abstract: In this talk we will discuss the recent solution of a question raised by K. Zhu about characterizing a class of singular integral operators on the Fock space. We show that for an entire function φ belonging to the Fock space $\mathscr{F}^2(\mathbb{C}^n)$ on the complex Euclidean space $\mathbb{C}^n$, the integral operator

$$S_\varphi F(z) = \int_{\mathbb{C}^n} F(w) e^{z\cdot\bar{w}} \varphi(z-\bar{w})\, d\lambda(w), \quad z \in \mathbb{C}^n,$$

is bounded on $\mathscr{F}^2(\mathbb{C}^n)$ if and only if there exists a function $m \in L^\infty(\mathbb{R}^n)$ such that

$$\varphi(z) = \int_{\mathbb{R}^n} m(x) e^{-2\left(x-\frac{i}{2}z\right)^2} dx, \quad z \in \mathbb{C}^n.$$

Here $d\lambda(w) = \pi^{-n} e^{-|w|^2} dw$ is the Gaussian measure on $\mathbb{C}^n$.

With this characterization we are able to obtain some fundamental results of the operator S_φ, including the normality, the C^* algebraic properties, the spectrum and its compactness. Moreover, we obtain the reducing subspaces of S_φ.

In particular, in the case $n = 1$, this gives a complete solution to the question proposed by K. Zhu for the Fock space $\mathscr{F}^2(\mathbb{C})$ on the complex plane $\mathbb{C}$ (Integr. Equ. Oper. Theory **81** (2015), 451–454).

This talk is based on joint work with Guangfu Cao, Ji Li, Minxing Shen, and Lixin Yan.