title: Dropping bodies to fill the Hill region

abstract: Drop N bodies, meaning, let them go from rest. Assuming that the only forces acting are the mutual gravitational attractions between masses, the resulting motion is a "brake orbit" solution to the N-body problem : a solution for which all velocities are zero at some instant. Such a solution has negative energy E and zero linear and angular momentum. Let $U > 0$ be the negative of the potential energy so that $E = K - U$ where $K \geq 0$ is the kinetic energy. All energy E solutions must lie within the Hill region $U \geq |E|$ of configuration space and all energy E brake orbits must touch the Hill boundary $U = |E|$. Our main result is that brake orbits sweep out the entire Hill region, including the collision locus $U = +\infty$. Our methods of proof are to combine the Jacobi-Maupertuis metric characterization of Newton's equations at constant energy with the direct method of the calculus of variations. Most of the ideas are contained in a 1948 paper of Seifert. (Seifert was known primarily as a topologist.) The new technical result we need to extend Seifert's methods so that they work in the non-compact setting of the N-body problem is that of a uniform 'metric collar' neighborhood of the Hill boundary, one 'uniformly' foliated by brake orbits.