

Title: *Towards the ultimate frontier of (finite dimensional) KAM Theory*

Abstract: In 2015, in occasion of the conference in honour of Amadeu, I announced a result, joint with Luca Biasco, which can be roughly rephrased as follows:

Theorem 1 *In bounded regions of phase space, except for a set of measure $\epsilon|\log \epsilon|^\gamma$, trajectories of nearly-integrable mechanical systems on $\mathbb{R}^n \times \mathbb{T}^n$ with generic real-analytic potentials of size $\epsilon \ll 1$ are quasi-periodic and span n -tori invariant for such systems.*

This theorem is in agreement (up to the logarithmic correction) with a conjecture formulated by Arnold, Kozlov and Neishtadt in the 2006 Springer Encyclopaedia of Mathematical Sciences (Chapter 6, p. 285).

A complete proof of the above result turned out to be much longer and more delicate than we thought and it has been completed only very recently, after a series of intermediate papers.

The purpose of my talk is to explain the precise results obtained recently, which include, as a particular case, the above theorem.