Possible contributed talk at CRM XVI Young Researchers Workshop in Geometry, Mechanics and Control

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Title
Geometric quantization via cotangent models.

Abstract

It is natural to ask for a universal model for geometric quantization associated to a real polarization given by an integrable system with non-degenerate singularities. In a joint work with Eva Miranda, we propose a universal model that goes one step further than the cotangent models in [KM17] by both considering singular orbits and adding to the cotangent models a model for the prequantum line bundle. These singularities are generic in the sense that are given by Morse-type functions and include elliptic, hyperbolic and focus-focus singularities. Examples of systems admitting such singularities are toric, semitoric and almost toric manifolds, as well as physical systems such as the coupling of harmonic oscillators, the spherical pendulum or the reduction of the Euler's equations of the rigid body on $T^*(SO(3))$ to a sphere. Our geometric quantization formulation coincides with the models given in [HM10] and [MPS20] away from the singularities and corrects former models for hyperbolic and focus-focus singularities cancelling out the infinite dimensional contributions obtained by former approaches. I will show that geometric quantization models provided here match the classical physical methods for mechanical systems such as the spherical pendulum as presented in [CS16]. Our cotangent models obey a *local-to-global principle* and can be glued to determine the geometric quantization of the global systems even if the global symplectic classification of the systems is not known in general.

On the other hand, and also concerning to geometric quantization, I will talk about equivalency of classical geometric quantization and formal geometric quantization, both for symplectic and b-symplectic manifolds. Finally, I will introduce the notion of Bohr-Sommerfeld leaves with sign in the geometric quantization of b-symplectic manifolds.

References

[CS16] Richard Cushman and Jedrzej Sniatycki. Classical and quantum spherical pendulum. *arXiv:1603.00966*, 2016.

[HM10] Mark D. Hamilton and Eva Miranda. Geometric quantization of integrable systems with hyperbolic singularities. *Ann. Inst. Fourier (Grenoble)*, 60(1):51–85, 2010.

[KM17] Anna Kiesenhofer and Eva Miranda. Cotangent models for integrable systems. *Comm. Math. Phys.*, 350(3):1123–1145, 2017.

[MPS20] Eva Miranda, Francisco Presas, and Romero Solha. Geometric quantization of almost toric manifolds. *J. Symplectic Geom.*, 18(4):1147–1168, 2020.