Reduction and Hamilton-Jacobi theory for systems with external forces

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Many physical systems require an external force, besides the Hamiltonian or Lagrangian function, to describe their dynamics. Moreover, external forces arise after performing a process of reduction in a Chaplygin nonholonomic system. Geometrically, external forces can be characterized as semibasic 1-forms on a symplectic manifold (usually, the phase space T^*Q or the space of velocities TQ). In our work, we extend significant results from geometric mechanics, such as Noether-like theorems, symplectic reduction or Hamilton-Jacobi theory, to systems with external forces.

In a first paper [1], we obtain a Noether's theorem for Lagrangian systems with external forces. We have also extended other results regarding symmetries and constants of the motion for systems with external forces. Our results are particularized for the so-called Rayleigh dissipation (i.e., an external force that can be written as the derivative of a "potential" with respect to the velocities). Furthermore, we present a theory for the reduction of Lagrangian systems with external forces, in which both the Lagrangian function and the external force are invariant under a Lie (sub)group of symmetries.

In a second paper [2], we develop a Hamilton-Jacobi theory for systems with external forces, particularizing for linear and non-linear Rayleigh systems, both in the Hamiltonian and Lagrangian frameworks. We study the complete solutions as well as their relation with constants of the motion in involution, and present some examples. We also present a dissipative bracket for Rayleigh forces. Additionally, making use of our results from the previous paper, we present a method for the reduction and reconstruction of solutions of the Hamilton-Jacobi problem for G-invariant forced Hamiltonian systems. Furthermore, we consider the reduction of a Čaplygin nonholonomic system to a system with external forces, so that one can obtain solutions of the Hamilton-Jacobi problem for the system with external forces and, from them, reconstruct solutions of the Hamilton-Jacobi problem for the nonholonomic system.

In a third paper [3], we develop a Hamilton-Jacobi theory for discrete forced Hamiltonian systems. Our approach is based in the construction of a discrete flow on $Q \times Q$ (unlike the conservative case, where the flow is defined on Q). In addition, we introduce the notion of a discrete Rayleigh potential and study its existence. Moreover, we present some simulations and analyse their numerical accuracy.

*Joint work with Manuel de León and Manuel Lainz; email: asier.lopez@icmat.es

References

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