An introduction to b-symplectic geometry and topology

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b^m-symplectic structures represent a construction analogous to standard symplectic geometry, incorporating a singular hypersurface (commonly denoted by Z) in which the "standard" symplectic structure blows up in a controlled way. These structures may appear in physical systems as non-canonical changes of coordinates are applied (see, for instance, the McGehee change of coordinates in the Restricted Circular Planar Three Body Problem). This kind of structures can be seen as a subset of Poisson structures, but they manifest a behaviour similar in its rigidity to that of symplectic manifolds. For instance, we have an analogous to the Moser path method, and therefore we have a Darboux theorem applicable in this context. The Poisson cohomology of b^m symplectic manifolds is also finite dimensional, as in the symplectic case, and it can be computed from the De Rham cohomology of both the base manifold and its singular hypersurface.

In order to build b-symplectic structures and to develop their properties analogous to the symplectic structures it is necessary to construct a vector bundle, ${}^bTM \to M$, called the *b-vector bundle*, which is locally isomorphic to TM everywhere in $M \backslash Z$. This raises the natural question of the classification of this vector bundle. Is it necessarily isomorphic to TM, or necessarily distinct? How does its isomorphy class relate to the topology of M, Z and the position of Z within M?

In this talk we are going to discuss briefly the basics of b-symplectic geometry, its most remarkable properties, and provide some insight on the questions related to the topology of the b-tangent bundle. We will also present several examples to yield some intuition on the expectations that we may have relating to this vector bundle.

The contents of this talk are based in a joint work with Eva Miranda.