Optimal Control of Mechanical System with Discretised Adjoint Methods

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Abstract

Optimal control theory has a broad range of applications in engineering and natural sciences such as pandemic modelling, aeronautics, or robotics, to name a few. When the physical problem is modelled with an initial or boundary value problem, the optimal problem may be found by solving a minimization or maximization of a cost functional subjected to ordinary or partial differential equations (ODEs or PDEs). Adjoint methods aim at solving the equations that arise from the optimality conditions of this optimization problem, which form a two-point boundary value problem (TPBVP). The analytical solution of these adjoint equations can only be solved in general for simple linear ODEs and linear or quadratic functional. We here propose a numerical discretization that allows solving non-linear ODEs and that guarantees the preservation of integrals of the continuous problem.

In this work we apply our methodology to a simple mass-spring mechanical system with Hamiltonian structure. We analyse two different strategies: (i) differentiate-discretise (d-D) and (ii) discretise-differentiate (D-d), also known as indirect and direct approaches, respectively. While the former leads to a discretized TPBVP, the latter results in a discrete initial value problem (IVP). In both cases, a non-linear programming problem needs to be solved, which has different properties depending on the strategy and discretization.

Here we analyse the mid-point time discretization (MPTD) schemes for modelling optimal control of the linear and nonlinear dynamical system with relatively larger time scales. We propose a local optimization problem is solved to populate the primal and dual variable trajectories as an initial guess for the global optimization problem. Control Hamiltonian preservation is used as key performance indicator to evaluate the algorithm robustness. It has been observed that the d-D approach with MPTD preserves Hamiltonian exactly for a linear dynamical system. Instead, the D-d approach with MPTD doesn't preserve Hamiltonian for the linear and nonlinear dynamical system. In future work, we will extend the current framework to design a structure-preserving algorithm to study the optimal control of worm locomotion on substrates with anisotropic friction.

Keywords— Optimal control theory, Control Hamiltonian, Time discretisation, Two-point boundary value problem, Dynamical system