Approximate XVA for European claims

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June 5, 2021

Abstract

We consider the problem of computing the Value Adjustment of European contingent claims when default of either party is considered, possibly including also funding and collateralization requirements. As shown in Brigo et al. in a series of papers (see Brigo D., Francischello M., Pallavicini A., *Nonlinear valuation under credit, funding, and margins: Existence, uniqueness, invariance, and disentanglement*, European Journal of Operational Research, (2019) and the references therein), this leads to a more articulate variety of Value Adjustments (the XVA's) that introduce some nonlinear features. When exploiting a reduced-form approach for the default times, the adjusted price can be characterized as the solution to a possibly nonlinear Backward Stochastic Differential Equation (BSDE). The expectation representing the solution of the BSDE is usually quite hard to compute even in a Markovian setting, and one might resort either to the discretization of the Partial Differential Equation characterizing it or to Monte Carlo Simulations. Both choices are computationally very expensive, in particular when considering stochastic default intensities. In this paper we suggest viewing such an expectation as a smooth function of the correlation parameters and to approximate it by its Taylor polynomial expansion around the zero vector (the independent case), in the hope that the first or second-order are enough to provide an accurate approximation. We apply our method to estimate the price contribution that comes from considering stochastic default intensities correlated with the underlying's price. We remark, though, that we can straightforward extend the same technique to include further stochastic factors. In order to evaluate Taylor polynomial's coefficients, we follow a two-step procedure to exploit, whenever possible, explicit formulae from option and bond's pricing theory. First, we condition the underlying's price with respect to the Brownian motions driving the intensities, retrieving a conditional explicit formula. Then, assuming the intensities to be described by affine models, we represent the single terms of the expansion using a change of Numéraire technique to disentangle the correlations between the asset's price and the default intensities. The affinity of the processes makes it possible to use a "bond-like" expression for the default component. The numerical discussion at the end of this work shows that, at least in the case of the Cox-Ingersoll-Ross (CIR) intensity model, even the simple first-order approximation has a remarkable computational efficiency.

Keywords: Pricing; Credit Value Adjustment; Defaultable Claims; XVA; Affine Processes.

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