

# THE VIX SMILE IN MULTI-FACTOR ROUGH VOLATILITY MODELS

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**Abstract.** We provide explicit small-time formulae for the at-the-money implied volatility of a large class of underlying assets, as well as its first and second derivatives with respect to the strike. Chiefly interested in VIX options, we draw insights from several past results arguing in favour of multi-factor stochastic (rough) volatility models, which are conducive to capture the smile behaviour. Therefore our asymptotic analysis includes underlyings measurable with respect to a multidimensional Brownian motion. Our proofs build on the framework of Alòs, Garcá-Lorite and Muguruza: they are based on Malliavin calculus techniques—in particular an anticipative Itô’s formula, and our formulae are expressed in terms of the Malliavin derivatives of the underlying.

We apply our general results to VIX options in a two-factor rough Bergomi model, with correlated Brownian motions, and derive closed-form expressions for the short-time at-the-money implied volatility level, skew, and curvature. These formulae only depend on the parameters of the model. Hence, they give analytical insights on the interplay between the different parameters, allowing to understand the behaviour of at-the-money VIX options prior to performing numerical tests. Nevertheless, to support our theoretical results, we also provide numerical examples and calibration with real data.

Finally, we present a similar analysis of the smile of the S&P index and, thanks to the additional degrees of freedom offered by the correlation parameters, we show how the two-factor rough Bergomi model succeeds in jointly calibrating the SPX and the VIX. Despite the extensive academic literature, options on VIX and S&P index still display different volatility surfaces, betraying the lack of a proper modelling framework. This issue is well-known as the SPX-VIX joint calibration problem and has motivated a number of creative modelling innovations in the past 15 years. Recently, the new paradigm of rough volatility has shown new promise in resolving this challenging problem.