High order approximations to call option prices in the Heston model.

Archil Gulisashvili[1], Marc Lagunas-Merino[2], Raúl Merino[3,4], and Josep Vives[3]

[1]Department of Mathematics, Ohio University, Athens OH 45701.
[2]Department of Mathematics, University of Oslo, P.O. Box 1053 Blindern, 0316 Oslo, Norway.
[3]Facultat de Matemàtiques, Universitat de Barcelona, Gran Via 585, 08007 Barcelona, Spain
[4]VidaCaixa S.A., Investment Risk Management Department, C/Juan Gris, 2-8, 08014 Barcelona, Spain.

June 2, 2021

Abstract

In this talk, a decomposition formula for the call price due to Alòs is transformed into a Taylor type formula containing an infinite series with stochastic terms. The new decomposition may be considered as an alternative to the decomposition of the call price found in a recent paper of Alòs, Gatheral and Rodoičić. We use the new decomposition to obtain various approximations to the call price in the Heston model with sharper estimates of the error term than in the previously known approximations. One of the formulas obtained in the present paper has five significant terms and an error estimate of the form $O(\nu^3(|\rho|+\nu))$, where ν and ρ are, respectively, the vol-vol an the correlation in the Heston model. Another approximation formula contains seven more terms and the error estimate is of the form $O(\nu^4\,(1+|\rho|\nu))$. For the uncorrelated Heston model ($\rho=0$), we obtain a formula with four significant terms and an error estimate $O(\nu^6)$. Numerical experiments show that the new approximations to the call price perform especially well in the high volatility mode.

References

E. Alòs (2012). *A decomposition Formula for Option Prices in the Heston Model and Applications to Option Pricing Approximation.* In: Finance and Stochastics 16 (3): 403-422.

E. Alòs, R. De Santiago and J. Vives (2015) *Calibration of stochastic volatility models via second-order approximation: the Heston case.* In: International Journal of Theoretical and Applied Finance 18 (6): 1550036 (31 pages).

E. Alòs, J. Gatheral and R. Radoičić (2020). *Exponentiation of Conditional Expectations Under Stochastic Volatility.* In: Quantitative Finance, Vol. 20, No. 1

S. L. Heston (1993). *A closed for solution for options with stochastic volatility with application to bond and currency options.* In: Review of Financial Studies 6: 293-326.

R. Merino, J. Pospíšil, T. Sobotka, and J. Vives (2018). *Decomposition formula for jump diffusion models.* In: International Journal of Theoretical and Applied Finance. Vol. 21, No. 08, 1850052.

A. Gulisashvili, R. Merino, M. Lagunas and J. Vives (2020). *Higher order approximation of call option prices under stochastic volatility models.* In: Journal of Computational Finance. Vol.n24, No. 1.