ON HOMOTOPY NILPOTENCY OF MOORE SPACE

MAREK GOLASIŃSKI

Given a group-like space X, we write $\varphi_{X,1} = \iota_X$, $\varphi_{X,2} : X \times X \to X$ for the basic commutator map and $\varphi_{X,n+1} = \varphi_{X,2} \circ (\varphi_{X,n} \times \iota_X)$ for $n \geq 2$. The *nilpotency class* $\operatorname{nil} X$ of the space X is the least integer $n \geq 0$ for which the map $\varphi_{X,n+1} \simeq *$ is nullhomotopic. If no such integer, we put $\operatorname{nil} X = \infty$.

With any based space X, we associate its loop space $\Omega(X)$, being a group-like space, to state

Theorem. 1. *Let $m \geq 1$, $n_1, \ldots, n_m \geq 2$ and $M(A_k, n_k)$ be Moore spaces of type (A_k, n_k) for $k = 1, \ldots, m$. Then:*

(1) $\operatorname{nil} \Omega((M(A_1, n_1) \times \cdots \times (M(A_m, n_m)) < \infty$ *if and only if if A_k are torsion-free groups with rank*

$\mathrm{r}(A_k) = 1$ *for $k = 1, \ldots, m$;*

(2) $\operatorname{nil} \Omega((M(A_1, n_1) \vee \cdots \vee M(A_m, n_m)) < \infty$ *if and only if $m = 1$ and A_1 is a torsion-free group*

with rank $\mathrm{r}(A_1) = 1$.

In particular, we derive

Corollary. 1. *If $M(A, n)$ is a Moore space with $n \geq 2$ then*

$$\operatorname{nil} \Omega(M(A, n)) < \infty$$

if and only if A is a torsion-free group with rank $r(A) = 1$ or equivalently, A is a subgroup of the rationals $\mathbb{Q}$.

Email address: marekg@matman.uwm.edu.pl

University of Warmia and Mazury, Olsztyn, Poland