A categorification of Yoshida's theorem for Mackey functors

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In the theory of Mackey functors for a finite group G, Yoshida's theorem provides an equivalence between Mackey functors which are *cohomological* (meaning that for all subgroups $K \leq H \leq G$ the composite $I_K^H R_K^H$ of restriction and induction is multiplication by the index $[H : K]$) with representations of the Hecke algebra $\mathcal{H}(G) = \mathrm{End}_{\mathbb{Z}G}(\bigoplus_{H \leq G} \mathbb{Z}[G/H])$. Typical examples of cohomological Mackey functors are group (co-) homology and Tate cohomology; the Burnside ring is a typical non-cohomological one. Reformulated in the "motivic picture" of Mackey functors, this says that if we quotient the category of spans of finite G-sets by the cohomological relations as above, we obtain the category of permutation G-modules.

In this talk I will present a categorification of Yoshida's theorem (see [2]), bearing on the *Mackey 2-functors* introduced in [1]. I will first define *cohomological* Mackey 2-functors, examples of which are given by abelian, derived or stable categories of linear group representations; a typical example of a non-cohomological Mackey 2-functor is given by the equivariant stable homotopy category. Our Yoshida theorem can be compactly formulated in the corresponding motivic picture, as saying that if we quotient the bicategory of *Mackey 2-motives* (constructed out of right-faithful spans of finite groupoids) by introducing the cohomological relations at the level of 2-cells, we obtain the bicategory of right-free permutation bimodules.

As motivation, I will explain the consequences this has for *block decompositions* of equivariant categories. We obtain for instance a direct and concrete way of comparing the usual blocks of representation theory (i.e. the primitive idempotents of the group algebra) and the additive decomposition of the equivariant stable homotopy category.

References

[1] Paul Balmer and Ivo Dell'Ambrogio. *Mackey 2-functors and Mackey 2-motives.* EMS Monographs in Mathematics. European Mathematical Society (EMS), Zürich, 2020.

[2] Paul Balmer and Ivo Dell'Ambrogio. Cohomological Mackey 2-functors. Preprint `https://arxiv.org/abs/2103.03974`, 2021.