$RO(C_2)$-graded coefficients of C_2-Eilenberg–MacLane spectra

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Computing the ordinary homology of a point in non-equivariant topology is a very easy task - we need only to take a look at the dimension axiom. The situation in G-equivariant topology is different. The G-equivariant version of the dimension axiom states that the ordinary homology graded over $\mathbb{Z}$ is concentrated in degree zero. However, the G-equivariant ordinary homology, known as the *Bredon homology*, is naturally graded over $RO(G)$ - the ring of G-representations, where the gradation over trivial representations corresponds to the $\mathbb{Z}$-grading mentioned in the dimension axiom. This makes the G-Bredon homology to have more “dimensions” - for example, the homology over C_2 is “two-dimensional”. During the talk I will present the method of computing the $RO(C_2)$-graded coefficients of C_2-Eilenberg–MacLane spectra based on the Tate square. Since the C_2-Eilenberg–MacLane spectra represent the C_2-Bredon homology, computing its coefficients is equivalent to computing the C_2-Bredon homology of a point. As demonstrated by Greenlees, the Tate square gives an algorithmic approach to computing the coefficients of equivariant spectra. In the talk we will discuss how to use this method to obtain the $RO(C_2)$-graded coefficients of a C_2-Eilenberg–MacLane spectrum as a $RO(C_2)$-graded abelian group. The big advantage of the method based on the Tate square is the possibility of keeping track of the mutiplicative structure. Basing on this, we will present the multiplicative structure of the C_2-Eilenberg- MacLane spectrum associated to the Burnside Mackey functor. This allows us to further describe the $RO(C_2)$-graded coefficients of any C_2-Eilenberg–MacLane spectrum as a module over the coefficients of the C_2-Eilenberg–MacLane spectrum of the Burnside Mackey functor. Finally, if the underlying Mackey functor is a ring, its C_2-Eilenberg–MacLane spectrum is a C_2-ring spectrum and thus its coefficients have the form of an $RO(C_2)$-graded ring. I will describe how to obtain this structure out of the Tate square. In particular, we will show that the coefficients of C_2-Eilenberg–MacLane spectra are always strictly commutative – i.e., the sign coming from the graded commutativity rule is always trivial.