Center of braid groups quotients and the homotopy groups of the 2-sphere

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One of the fundamental problems in algebraic topology is to study the homotopy groups of spheres, and in general they are unknown. Let $P_n(M)$ denote the pure braid group with n strings over a surface M. Let $Z(G)$ denote the center of a given group G. In this work we will show that, for $n \geq 3$ and M being the disk D^2 or the 2-sphere S^2, there exists a normal subgroup $G_n(M)$ of $P_n(M)$ such that

$$Z\left(\frac{P_n(D^2)}{G_n(D^2)}\right) \cong \pi_n(S^2) \times \mathbb{Z}$$

and

$$Z\left(\frac{P_{n+1}(S^2)}{G_{n+1}(S^2)}\right) \cong \pi_n(S^2) \times \mathbb{Z}_2.$$

The group $G_n(M)$ can be explicitly described by iterated commutators using the standard Artin generators for pure braids.

For $M = D^2$ this result is due to J.Y.Li and J.Wu [Proc. London Math. Soc. - 2009], however we give a different proof for this case. The case $M = S^2$ is new. Moreover, the ideas developed here works for pure braid groups over any surface.