Excursions in Mathematical Finance

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The risk and return profiles of a broad class of dynamic trading strategies –including pairs trading and other statistical arbitrage strategies – may be characterized in terms of excursions of the market price of a portfolio away from a reference level. We propose a mathematical framework for the modelling and risk analysis of such strategies, based on a description in terms of *price excursions*.

In these lectures we develop these ideas, first in a pathwise setting without probabilistic assumptions, then in a Markovian setting. We connect this viewpoint with the beautiful theory of excursions of stochastic processes, and show how concepts from excursion theory are useful for the analysis of dynamic trading strategies.

We introduce the notion of δ-excursion, defined as a path which deviates by δ from a reference level before returning to this level. We show that every continuous path has a unique decomposition into δ-excursions, which is useful for scenario analysis of dynamic trading strategies, leading to simple expressions for the number of trades, realized profit, maximum loss and drawdown of dynamic trading strategies. As δ is decreased to zero, properties of this decomposition relate to the local time of the path, linking the high-frequency asymptotics of such strategies to the roughness of the price path.

When the underlying asset follows a Markov process, we combine these results with Ito's excursion theory [3] to obtain a tractable decomposition of the process as a concatenation of independent δ-excursions, whose distribution is described in terms of Ito's excursion measure. We provide analytical results for linear diffusions and give new examples of stochastic processes for flexible and tractable modeling of excursions. Finally, we describe a non-parametric scenario simulation method for generating paths whose excursion properties match those observed in empirical data.

References

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