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Invariant components of the Fatou set

Singular orbits and Baker domains

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Singular orbits and Baker domains

Lasse Rempe

Department of Mathematical Sciences, University of Liverpool

Topics in Complex Dynamics, Barcelona, April 2021

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Invariant Fatou components of rational maps

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- *Fatou set F*(*f*) set of normality;
- A *Fatou component* is a connected component of *F*(*f*).
- A Fatou component *U* is *invariant* if $f(U) \subset U$.

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Types of invariant Fatou components

Every invariant Fatou component of a *rational map* is one of the following:

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Types of invariant Fatou components

Every invariant Fatou component of a *rational map* is one of the following:

- an immediate (super-)attracting basin;
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Types of invariant Fatou components

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Types of invariant Fatou components

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(Image by Arnaud Chéritat)

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By work of *Fatou*, there is a close relationship between invariant Fatou components of a rational map *f* and the *critical values* of *f*:

The role of critical values



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By work of *Fatou*, there is a close relationship between invariant Fatou components of a rational map *f* and the *critical values* of *f*: • Every attracting or parabolic basin *contains a critical value*;

• The boundary of any rotation domain is in the postcritical set

The role of critical values

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The role of critical values

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- Every attracting or parabolic basin contains a critical value;
- The boundary of any rotation domain is in the *postcritical set*

$$\mathcal{P}(f) := \overline{\bigcup_{f'(c)=0} \bigcup_{n=1}^{\infty} f^n(c)}.$$

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By work of *Fatou*, there is a close relationship between invariant Fatou components of a rational map *f* and the *critical values* of *f*:



The role of critical values

(Image by Arnaud Chéritat)

Meromorphic functions

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Now let

$f\colon \mathbb{C}\to \hat{\mathbb{C}}$

be a transcendental meromorphic function.

- *Fatou set* F(f): Iterates are *defined* and form a normal family.
- Note: All *poles* and their preimages under *fⁿ* are in the *Julia set* J(f) = C \ F(f).

Meromorphic functions

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Meromorphic functions

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Singular values

$f \colon \mathbb{C} \to \hat{\mathbb{C}}$ transcendental meromorphic.

The role of critical values is now played by

sing(f^{-1}) = {"singularities of f^{-1} "} = {critical values} \cup {asymptotic values}.

Postsingular set:

$$\mathcal{P}(f) := \overline{\bigcup_{n=0}^{\infty} f^n(\operatorname{sing}(f^{-1}))}.$$

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Invariant Fatou components

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- an immediate (super-)attracting basin;
 (intersects sing(f⁻¹))
- an immediate parabolic basin;
 (intersects sing(f⁻¹))
- a *rotation domain*. (boundary in $\mathcal{P}(f)$)
- a *Baker domain U*, where $f^n(z) \to \infty$ for $z \in U$.

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Invariant Fatou components

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Every invariant Fatou component of a *transcendental meromorphic function* is one of the following:

 $z \mapsto \sin(z)/2$



attracting basin

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parabolic basin

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Every invariant Fatou component of a *transcendental meromorphic function* is one of the following:



Siegel disc

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Invariant Fatou components

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Baker domain

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Singular values and Baker domains

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In his famous 1993 survey on the *iteration of meromorphic functions*, Bergweiler asks:

Question 4. Let f be a meromorphic function with a cycle of Baker domains that does not contain a point of $sing(f^{-1})$. Is there some relation between $sing(f^{-1})$ and the boundaries of the domains of this cycle?

In particular, he asks:

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In particular, he asks:

Question 5. Is it possible that a meromorphic function f has Baker domains if $O^+(z)$ is bounded for all $z \in sing(f^{-1})$?

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A Baker domain despite finite singular orbits

Theorem (R., 2021)

There is a transcendental meromorphic function f such that every point of $sing(f^{-1})$ is a superattracting periodic point of period 2, and such that f has an invariant Baker domain.

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Postsingular points near Baker domains

Concerning *Question 4*, Bergweiler showed in 1995 that there exists a transcendental *entire* function *f* having a Baker domain *U* such that $dist(\mathcal{P}(f), U) > 0$. On the other hand, he proved the following.

Theorem (Bergweiler, 1995)

Let f be a transcendental entire function with a Baker domain U disjoint from $sing(f^{-1})$. Then there is a sequence (p_n) in $\mathcal{P}(f)$ such that

(a)
$$|p_n| \to \infty;$$

(b) $\frac{\operatorname{dist}(p_n, U)}{|p_n|} \to 0; an$

(c)
$$\lim_{n\to\infty} \frac{p_{n+1}}{p_n} = 1$$
.

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Theorem (Mihaljević-R., 2013)

The same holds for meromorphic f provided that we replace (c) by

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Bergweiler (f entire):

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Question (Mihaljevi'c-R., 2013)

Can (**) be replaced by (*) for meromorphic f?

Theorem (Barański-Fagella-Jarque-Karpińska, 2020)

Yes, if $\mathbb{C} \setminus U$ has an unbounded component.

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Theorem (R., 2021)

There is a transcendental meromorphic function f such that every point of $sing(f^{-1})$ is a superattracting periodic point of period 2, and such that f has an invariant Baker domain.

Given ho > 0, the function f can be chosen such there is a sequence ${f R}_j o \infty$ with

 $\{z\in\mathbb{C}\colon R_j<|z|<\rho R_j\}\subset U$

for all j.

The proof uses quasiconformal surgery, starting from the linear map

 $z\mapsto \mu z,$

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where $|\mu| > 1$.

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