Topological and Combinatorial Dynamics

Centre de Recerca Matemàtica

April 6th to 9th, 2021

Abstracts Book

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Abstracts

of the

Invited talks

On circle diffeomorphisms driven by strongly expanding circle maps

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We present some results about the dynamics of circle diffeomorphisms which are driven by strongly expanding endomorphisms of the circle.

On typical properties of Lebesgue measure preserving interval/circle maps

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We survey the older and also recent results concerning typical properties of Lebesgue measure preserving interval/circle maps. We show that the typical continuous maps of the interval which preserve the Lebesgue measure satisfies the shadowing and periodic shadowing properties and for each $k \geq 1$ the set of periodic points of period k is a Cantor set of Hausdorff dimension zero and upper box dimension one. We also show that typical Lebesgue measure preserving circle maps have the s-limit shadowing property. In addition we obtain that s-limit shadowing is a typical property also for continuous circle maps. In particular, this implies that classical shadowing, periodic shadowing and limit shadowing are typical in the respective two settings as well.

Acknowledgments: The speaker was supported by the European Regional Development Fund, project No. CZ 02.1.01/0.0/0.0/16_019/0000778.

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Dilation surfaces and affine interval exchanges

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This talk will deal with dilation surfaces and affine interval exchanges. These objects are natural extensions of the well known translation surfaces and classic interval exchanges. A dilation surface is a punctured surface which carries an atlas of charts whose changes of coordinates are dilation maps of the form

$$z \mapsto az + b$$

where $a \in \mathbb{R}_+^*$ and $b \in \mathbb{C}$.

These surfaces, like translation surfaces, come with a natural foliation, called the vertical foliation, defined in any charts by the equation $x = \text{cst}$ (invariant by change of coordinates).

First return maps of these foliations are piecewise affine maps so that their dynamical properties are closely related to those of affine maps.

The talk will focus on introducing the specificities of this setting and on surveying the few that is known.

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Discrete Melnikov functions with applications

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We start studying the number of periodic solutions of linear, Riccati and Abel periodic differential or difference equations. We recover some known results for the corresponding differential equations and obtain new ones for the difference equations. In particular, we prove that there is no upper bound for the number of isolated periodic solutions of Abel difference equations by using a suitable discrete Melnikov function. This approach motivates us to introduce the first order Melnikov functions for more general periodic discrete dynamical systems.

More concretely, we consider non-autonomous N-periodic d-dimensional discrete dynamical systems of the form $r_{n+1} = F_n(r_n, \varepsilon)$, having when $\varepsilon = 0$ an open continuum of initial conditions such that the corresponding sequences are N-periodic. From the study of some variational equations of low order we obtain successive maps, that we call discrete Melnikov functions, such that the simple zeroes of the first one that is not identically zero control the initial conditions that persist as N-periodic sequences of the perturbed discrete dynamical system. We apply these results to several examples, including some non-autonomous perturbed globally periodic difference equations. Talk based on the works [1, 2].

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Topological models of zero entropy loosely Bernoulli systems

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We provide a purely topological description of minimal and uniquely ergodic systems whose unique invariant measure is loosely Bernoulli and has zero entropy (we call such measure preserving systems *loosely Kronecker*). At the heart of our result lies Feldman-Katok continuity, that is, continuity with respect to the recently introduced Feldman-Katok pseudometric (see [2]), which is a topological counterpart of the pseudometric f-bar on a symbolic space. The talk is based on [1].

Acknowledgments: The research of DK was supported by the National Science Centre, Poland, grant no. 2018/29/B/ST1/01340.

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Periods of continuous maps on some compact spaces

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The objective of this talk is to provide information on the set of periodic points of a continuous self-map defined in the following compact spaces: the n-dimensional sphere, the product space of an n-dimensional with an m-dimensional spheres, the n-dimensional complex projective space and the n-dimensional quaternion projective space. We use as main tool the action of the map on the homology groups of these compact spaces and the Lefschetz zeta-function.

Piecewise Homeomorphisms of the circle, a geometrization result.

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In this talk I will describe a recent result relating dynamics of some piecewise homeomorphisms of the circle with geometric group theory. The goal is to construct a group from the dynamics of a particular class of maps and then study this group. In this very particular class of maps we prove that the group in question is a Gromov hyperbolic surface group witch is related with the dynamics by a strong property : an Orbit Equivalence. Such maps are called Bowen-Series like maps.

Topological entropy of Bunimovich stadium billiards

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We estimate from below the topological entropy of Bunimovich stadium billiards. We do it for long billiard tables, and find the limit of estimates as the length goes to infinity.

On the Entropy Conjecture of Marcy Barge

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I shall discuss a positive solution to the following problem.

Question (M. Barge, 1989 [8]) Does there exist, for every $r \in [0, \infty]$, a pseudo-arc homeomorphism whose topological entropy is r?

Until now all known pseudo-arc homeomorphisms have had entropy 0 or ∞. Recall that the pseudo-arc is a compact and connected space (continuum) first constructed by Knaster in 1922 [6]. It can be seen as a pathological fractal. According to the most recent characterization [5] it is topologically the only, other than the arc, continuum in the plane homeomorphic to each of its proper subcontinua. The pseudo-arc is homogeneous [2] and played a crucial role in the classification of homogeneous planar compacta [4]. Lewis showed that for any n the pseudo-arc admits a period n homeomorphism that extends to a rotation of the plane, and that any P-adic Cantor group action acts effectively on the pseudo-arc [7] (see also [10]). We adapt Lewis' inverse limit constructions, by combining them with a Denjoy-Rees scheme [1] (see also [9], [3]). The positive entropy homeomorphisms that we obtain are periodic point free, except for a unique fixed point.

I am going to present various results related to the problem, to conclude with a discussion of its solution.

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Product of Minimal Spaces: Complete Solution

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A metric space is said to be a *minimal space*, if it admits a minimal (not necessarily invertible) map. The classification of minimal spaces is a well-known open problem in topological dynamics, solved only in some particular cases; for some references see e.g. [6].

Even such a basic and natural *question*, explicitly posed in [2, p. 126], as whether the product of two compact minimal spaces is a minimal space, has not been answered so far in its full generality, though recently two results related to the question have appeared:

- The class of compact metric spaces admitting minimal continuous *flows* is closed with respect to at most countable products, see [3, Theorem 25].
- In the special case when homeomorphisms rather than continuous maps are considered, a negative answer has been provided. There is a metric continuum Y admitting a minimal *homeomorphism* such that $Y \times Y$ does not admit any minimal *homeomorphism* [1] (the space Y is obtained as a modification of a space from [5]).

We solve the problem completely:

- The answer to the question is negative, i.e. the product of minimal spaces need not be minimal [4, Theorem A].

We also add several related results and we formulate an open problem.

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Universality, self-similarity and a renormalization operator for 1D maps under quasiperiodic forcing

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We consider an extension of the classical one dimensional renormalization operator to the case where the one-dimensional map is forced quasiperiodically. One paradigmatic example is the quasiperiodically forced logistic map (FLM). In the bifurcation diagram of the one-dimensional Logistic map it is well known that there exist parameter values for which the 2^n periodc orbit is superattracting. In the quasiperiodically case these orbits are replaced by invariant curves, that undergo a finite sequence of period doublings. By means of an extension of the classic one-dimensional renormalization operator to the quasiperiodic case it is possible to understand the presence of self-similarities in the bifurcation diagram of the invariant curves of the FLM. In this talk we numerically describe a self-similarity in the bifurcation diagram of the FLM associated to the curves of reducibilty loss (corresponding to the passage from a reducible to non-reducible invariant curve of a 2^n periodic invariant curve). The study of the dynamics of the renormalization operator allow us to give an interpretation of this self-similar behaviour (see [1, 2, 3]).

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Abstracts of the Contributed Talks

Remarks on homoclinic orbits

Francisco Balibrea

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In $f \in C^0(I, I)$ where $I = [0, 1]$, is well known three equivalences; that f has a periodic point of period not a power of two, the topological entropy of f is positive and f has a *homoclinic point*. A homoclinic point implies a structure on I which provokes the complexity on the dynamics of f. For example if f has a homoclinic point then $\lim_{n\to\infty} V_{\overline{P(f)}}(f^n) = \infty$, where $V(f^n)$ denotes the total variation of the iterates of f. There are another new results for interval maps connected with the appearence of *distributional chaotic points of type 1, 2 and 3* and the condition $h(f) > 0$.

Part of the above results can be stated for continuous maps on I^2, particularly inside the class of *triangular maps*. The aim of this talk is to deal with some of the above results and state some open questions.

Topological degree and periodic orbits of semi-dynamical systems

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We study semi-dynamical systems associated to delay differential equations. With population models in mind [1], we consider the delayed differential system

$$x'(t) = f(t, x(t), x(t - \tau)) \tag{1}$$

where $f : R \times [0, +\infty)^{2N} \longrightarrow R^N$ is continuous and $\tau \in \mathbb{R}^+$ is the delay. An initial condition for (1) can be expressed in the following way

$$x_0 = \varphi, \tag{2}$$

where $\varphi : [-\tau, 0] \longrightarrow [0, +\infty)^N$ is a continuous function and $x_t \in C([-\tau, 0], \mathbb{R}^N)$ is defined by $x_t(s) = x(t + s)$. Thus, the flow

$$\Phi : [0, +\infty) \times C([-\tau, 0], \mathbb{R}^N) \longrightarrow C([-\tau, 0], \mathbb{R}^N) \tag{3}$$

given by $\Phi(t, \varphi) = x_t$, induces a semi-dynamical system.

In order to find periodic orbits of (3) we employ topological degree methods. Since the space of initial conditions is infinite dimensional, the Brouwer degree cannot be applied: we use instead Leray–Schauder degree techniques. More precisely, inspired by [4], we shall work on the positive cone X of C_T, the Banach space of continuous T-periodic functions, for some $T > 0$, and define an appropriate fixed point operator $K : X \longrightarrow C_T$. We observe that if f is T-periodic in the first coordinate, then K is well defined and its fixed points determine T-periodic solutions of equation (1).

To this end, we show that the system has fixed points via proving that the Leray–Schauder degree of the operator $I - K$ is nonzero on an appropriate subset $U \subset X$. Indeed, we are able to construct an homotopy between $I - K$ and $I - K_0$, with K_0 a compact operator such that its image is contained in the subset of constant functions in $\Omega := U \cap \mathbb{R}^N$ and, in turn, $I - K_0$ is homotopic to the inward normal on Ω. We then apply the Hopf theorem (see e.g. [3]): if the Euler characteristic of Ω is nonzero, then the degree of the inward normal is nonzero.

The results are part of the work [2].

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An Algorithm to Compute rotation numbers in the circle

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We present an efficient algorithm to compute rotation numberer of a faimily of cirlce maps of degree one. This family, which we shall call $\mathcal{L}_1^{ccs}$ corresponds to the non-decreasing maps that have at least a constant section $[\alpha, \beta]$ and a lifted cycle P with $P \cap [\alpha, \beta] \neq \emptyset$. This algorithm can be used to compute the rotation interval of general maps of degree one.
We also compare our algorithm with existing ones by ploting the Devil's Staircase using all the tested algorithms and comparing the results and the time they take.
Finally, using the proposed algorithm we will plot the Arnold Tongues and rotation intervals of some functions, most of which cannot be quickly computed with the existing algorithms.

The parameterization method for invariant curves of parabolic points

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We consider a map F of class C^r with a fixed point of parabolic type whose differential is not diagonalizable, and we study the existence and regularity of the invariant manifolds associated with the fixed point using the parameterization method [1]. Concretely, we show that under suitable conditions on the coefficients of F, there exist invariant curves of class C^r away from the fixed point, and that they are analytic when F is analytic. The differentiability result is obtained as an application of the fiber contraction theorem. We also provide an algorithm to compute an approximation of a parameterization of the invariant curves and a normal form of the restricted dynamics of F on them. The talk is based on the work [2].

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Complex Trees: Topological Combinatorics of Connected Self-similar Sets

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The theory of complex trees is introduced as a new approach to study a broad class of self-similar sets which includes Cantor sets, Koch curves, Lévy C curves, Sierpinski gaskets, Rauzy fractals, plane-filling curves, and fractal dendrites. We note a fundamental dichotomy for n-ary complex trees that allows us to study topological changes in regions where one-parameter families of connected self-similar sets are defined. Moreover, we show how to obtain these families from systems of equations encoded by tip-to-tip equivalence relations. The parameter space maps that we introduce to study these families of connected self-similar sets are new. And for $T_A(z) := T\{z, \frac{1}{2}, \frac{1}{4z}\}$ we show that the boundary surrounding structurally stable trees is piecewise smooth.

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Entropy beyond actions of uniform lattices

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Measure theoretical entropy is an intensely studied concept with various applications and interpretations. For actions of non-discrete groups, such as $\mathbb{R}^d$ it can be defined by computing the entropy with respect to a uniform lattice, such as $\mathbb{Z}^d$. Nevertheless, there exist (metrizable and σ-compact) locally compact Abelian groups, such as the additive group of p-adic numbers, that do not contain uniform lattices.

In this talk we explore two non-equivalent notions of entropy, which both generalize the notion of entropy from the setting of discrete amenable groups to the setting of unimodular amenable groups. The first concept is defined by using the concept of (thin) Følner nets from [1]. The second concept will be defined by replacing the uniform lattice by a weaker structure, called a Delone set, which exists in every unimodular amenable group. This concept generalizes the concept considered in [2]. We relate these notions to the respective notions of topological pressure, present a link to naive entropy and proof respective versions of Goodwyn's half of the variational principle.

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