IRP LDDS

Transcendental dynamics and beyond: Topics in Complex Dynamics

Online via Zoom

From April 19$^{\text{th}}$ to April 23$^{\text{rd}}$, 2021

Abstracts Book

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10:00 – 10:50	**On how to use box mappings as "black boxes"** Kostya Drach, *Aix-Marseille University*
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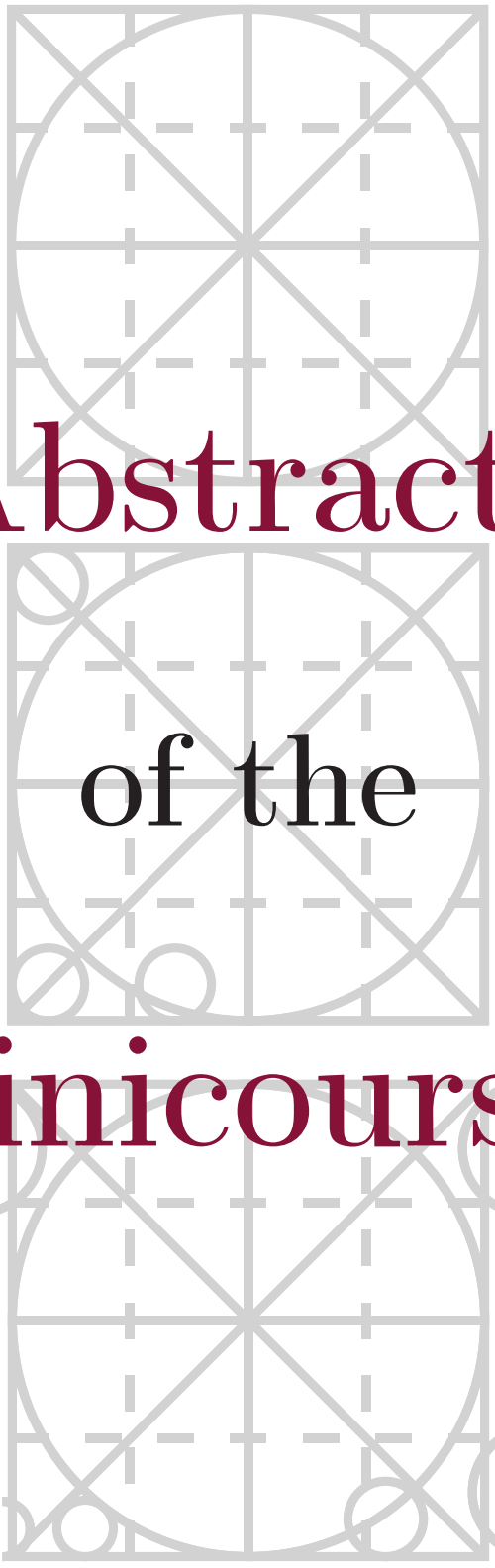
12:10 – 13:00	**On the dimension of escaping sets for exponentials** Bogusława Karpińska, *Warsaw University of Technology, Faculty of Mathematics and Information Science*



Abstracts of the Minicourses

Dynamics of transcentental Henon maps

Anna Benini

Università di Parma, Italy.

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We will outline the main features of dynamics in C^2 as opposed to dynamics over the complex plane. We will then explain some dynamical features of a special class of non-polynomial automorphisms of C^2, called transcendental Henon maps, highlighting similarities with the dynamics of transcendental entire functions in one variable.

Trees, triangles and tracts

Christopher Bishop

Stony Brook University, New York, United States of America.

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These two lectures will motivate, state and apply a method for constructing holomorphic functions called quasiconformal folding. In the first lecture, I will review the definition and some properties of harmonic measure on planar domains and consider the problem of constructing conformally balanced trees, i.e., planar trees for which every edge gets equal harmonic measure and the two restrictions of harmonic measure to each side of an edge are equal. Such "true trees" are a special case of Grothendieck's theory of "dessins d'enfants" and correspond in a natural way to polynomials with exactly two critical values. We will address the questions of which trees can be drawn in this way, and what are the possible shapes of such trees. We will end with some connections to polynomial dynamics and Riemann surfaces.

The second lecture considers the analogous connection between infinite planar trees and entire functions with two singular values. I will state the Folding Theorem, which takes an infinite planar tree with certain geometric assumptions, and returns an entire function with two singular values. I will sketch the idea of the proof, and then describe some applications of the folding construction to transcendental dynamics, e.g., the construction of wandering domains in the Eremenko-Lyubich class, and the construction of transcendental Julia sets with dimension close to 1 (all transcendental Julia sets have dimension at least 1).These two lectures will motivate, state and apply a method for constructing holomorphic functions called quasiconformal folding. In the first lecture, I will review the definition and some properties of harmonic measure on planar domains and consider the problem of constructing conformally balanced trees, i.e., planar trees for which every edge gets equal harmonic measure and the two restrictions of harmonic measure to each side of an edge are equal. Such "true trees" are a special case of Grothendieck's theory of "dessins d'enfants" and correspond in a natural way to polynomials with exactly two critical values. We will address the questions of which trees can be drawn in this way, and what are the possible shapes of such trees. We will end with some connections to polynomial dynamics and Riemann surfaces.

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Harmonic and Subharmonic Functions

Phil Rippon and Gwyneth Stallard

Open University, United Kingdom.

In the first lecture we'll give an introduction to the basic properties of harmonic and subharmonic functions and some key results, including the Dirichlet problem, Harnack's inequality, the maximum principle, harmonic measure and estimates for harmonic measure.

In the second lecture we'll show how these properties of harmonic and subharmonic functions provide useful tools in proving results in complex dynamics, using examples such as properties of escaping wandering domains, covering properties of annuli, and analysing dynamical behaviour in multiply connected wandering domains.

Abstracts of the Invited Speakers

TBA

Matthieu Astorg

Institut Denis Poisson, France.

To be announced.

Typical behaviour of boundary points of Fatou components of transcendental maps

Krzysztof Barański

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We study typical behaviour of points in the boundaries of periodic Fatou components of transcendental entire or meromorphic maps. We show that for a simply connected invariant Baker domain U of a meromorphic map f with a finite degree on U, either typical boundary points (with respect to harmonic measure) escape to infinity or typical points have dense trajectories in the boundary of U, depending on the Baker–Cowen–Pommerenke type of U. If U is parabolic, only the second case occurs. Furthermore, if U is an attracting periodic component of period larger than 1 of an exponential map, its typical boundary points (in the sense of dimension) do not escape to infinity. The talk is based on joint results with N. Fagella, X. Jarque and B. Karpińska [1] as well as B. Karpińska and A. Zdunik [2].

References

[1] Krzysztof Barański, Núria Fagella, Xavier Jarque and Bogusława Karpińska, *Escaping points in the boundaries of Baker domains*, J. Anal. Math. **137.2** (2019), 679–706.

[2] Krzysztof Barański, Bogusława Karpińska and Anna Zdunik, *Dimension properties of the boundaries of exponential basins*, Bull. Lond. Math. Soc. **42.2** (2010), 210–220.

On the geometry of simply connected wandering domains

Luka Boc Thaler

University of Ljubljana, Slovenia.

In this talk we will construct an entire function $f : \mathbb{C} \to \mathbb{C}$ for which the unit disk $\mathbb{D}$ is an escaping wandering domain. The construction relies on the approximation techniques and can be generalized so that the unit disk $\mathbb{D}$ is replaced by any bounded connected regular open set U, whose closure has a connected complement. In particular this implies that every simply connected Jordan domain is an escaping wandering domain of some entire function. Some additional work is needed to prove that the word "escaping" can be replaced by "oscillating", see [2]. Similar approach was used in [1] to show that a unit ball $\mathbb{B} \subset \mathbb{C}^m$ is a wandering domain of some automorphism of $\mathbb{C}^m$.

References

[1] Luka Boc Thaler, *Automorphisms of $\mathbb{C}^m$ with bounded wandering domains*, Annali di Matematica Pura ed Applicata (2021), doi: 10.1007/s10231-020-01057-3

[2] Luka Boc Thaler, *On the geometry of simply connected wandering domains*, Preprint: arXiv:2012.13284

On how to use box mappings as "black boxes"

Kostya Drach

(in collaboration with Trevor Clark, Oleg Kozlovski, Dierk Schleicher and Sebastian van Strien)

Aix-Marseille University, France.

The concept of a complex box mapping is a generalization of the classical notion of polynomial-like map to the case when one allows for countably many components in the domain and finitely many components in the range of the mapping. In one-dimensional dynamics, box mappings appear naturally as first return maps to certain nice sets. In this talk, we will discuss various features of general box mappings, as well as so-called dynamically natural box mappings, focusing on the irrigidity and ergodicity properties. We will then show how these results can be used almost as "black boxes" to conclude similar properties in those families of rational maps where non-trivial box mappings can be extracted. The most prominent illustrative examples for us will be complex polynomials of arbitrary degree and their Newton maps. The construction in the polynomial case is standard and uses Yoccoz puzzles. In the Newton case, the construction of box mappings relies on the recently developed notion of Newton puzzles.

A Newhouse Phenomenon in Trancendental Dynamics

Adam Epstein
(in collaboration with Lasse Rempe)

Warwick University, United Kingdom.

Jointly with Lasse Rempe, we prove the existence of entire functions in the Eremenko–Lyubich class with infinitely many attractors.

Boundary dynamics of simply connected wandering domains

Vasiliki Evdoridou

(in collaboration with A.M. Benini, N. Fagella, P. Rippon and G. Stallard)

Open University, United Kingdom.

Wandering domains of transcendental entire functions, i.e. Fatou components that are not eventually periodic, have been extensively studied in recent years. For example, a nine-way classification of the internal dynamics in simply connected wandering domains has been given. In this talk we focus on the dynamical behaviour on the boundaries of simply connected wandering domains. In particular, we consider the possibility that most boundary orbits converge together in a certain sense, and give sufficient conditions for such a convergence to hold. Our results are motivated by and extend classical results on the boundary dynamics of inner functions.

On the dimension of escaping sets for exponentials

Bogusława Karpińska

(in collaboration with Krzysztof Barański)

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We determine the Hausdorff and packing dimension of sets of points which escape to infinity at a given rate under non-autonomous iteration of exponential maps. In particular we generalize the results proved by Sixsmith in 2016.

Interpolation of Power Mappings

Kirill Lazebnik
(in collaboration with Jack Burkart)

University of Toronto, Toronto, Canada.

Let $(M_j)_{j=1}^{\infty} \in \mathbb{N}$ and $(r_j)_{j=1}^{\infty} \in \mathbb{R}^+$ be increasing sequences satisfying some mild rate of growth conditions. We prove that there is an entire function $f : \mathbb{C} \to \mathbb{C}$ whose behavior in the large annuli $\{z \in \mathbb{C} : r_j \cdot \exp(\pi/M_j) \leq |z| \leq r_{j+1}\}$ is given by a perturbed rescaling of $z \mapsto z^{M_j}$, such that the only singular values of f are rescalings of $\pm r_j^{M_j}$. We describe several applications to the dynamics of entire functions.

On the computational complexity of Julia sets in the exponential family

David Martí-Pete

(in collaboration with Artem Dudko)

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Computer generated images of Julia sets play a crucial role in establishing new results in complex dynamics. Roughly speaking, a subset of a plane is called computable if there is an algorithm that can produce an approximation of this set with arbitrarily high precision. Computational complexity measures how long it takes for such an algorithm to produce an approximation with a given precision. In this talk, we will give an overview of the main results in this research area, and then we will focus on the computability of Julia sets of functions in the exponential family that have an attracting cycle. These include functions for which the Julia set consists of an uncountable collection of unbounded disjoint curves known as a Cantor bouquet.

Connectivity and unboundedness of Fatou components for elliptic functions

Mónica Moreno Rocha

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A natural consequence of the periodicity of an elliptic function is the invariance of its Fatou and Julia sets under the action of the lattice. This invariance has an important effect on the unboundedness of Fatou components. In turn, every example of unbounded Fatou components are known to be infinitely connected, while bounded ones were known, until recently, to be simply connected. In this talk I will outline the existence of Herman rings for elliptic functions, thus showing that doubly-connected bounded components are possible. After explaining the mechanisms that produce an unbounded Fatou component, I will address the question regarding the existence of finitely connected unbounded components.

Iterating the minimum modulus

Daniel Nicks

(in collaboration with Gwyneth Stallard and Phil Rippon)

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For an entire function f there may or may not exist an $r > 0$ such that the iterated minimum modulus $m^n(r) \to \infty$ as $n \to \infty$. Here $m(r) = m(r, f) = \min\{|f(z)| : |z| = r\}$. Focussing mainly on the class of real transcendental entire functions of finite order with only real zeroes, we discuss some results about the existence of an $r > 0$ such that $m^n(r) \to \infty$. This is motivated by the result that, for functions in this class, the existence of such an r implies that the escaping set of f is connected, which confirms Eremenko's conjecture under these conditions.

A Central Limit Theorem for Inner Functions

Artur Nicolau
(in collaboration with Odi Soler)

Universitat Autònoma de Barcelona, Spain.

A Central Limit Theorem for linear combinations of iterates of an inner function will be presented. The main idea is that roughly speaking, iterates of an inner function behave as independent random variables.The main technical tool is Aleksandrov Desintegration Theorem for Aleksandrov-Clark measures.

Using complex dynamical methods to describe the computational complexity of approximating the independence polynomial

Han Peters

University of Amsterdam, Netherlands.

The independence polynomial arises in statistical physics as the partition function of the Hard-core model. Its value in the value 1 counts the number of independent subsets of a graph, a computationally hard problem. In fact, exact computation of the independence polynomial is hard for most parameter values.

Instead of exact computation, one can look for efficient algorithms to approximately compute values of the independence polynomial. In current joint work with David de Boer, Pjotr Buys, Lorenzo Guerini and Guus Regts, all from the University of Amsterdam, we characterize the parameters for which approximation is hard: the closure of these parameters contains the closure of the zero-locus, which in turn coincides with the activity locus of a related family of rational functions.

For graphs of large bounded degrees we can give a more precise description of the domain where effective algorithms exist. Our method relies upon classical results in transcendental dynamics. In this talk I will focus on the relationship with complex dynamical systems.

Singular orbits and Baker domains

Lasse Rempe

University of Liverpool, United Kingdom.

Fatou showed that there is a close relationship between invariant components of the Fatou set of a rational function and its critical values. Indeed, every attracting or parabolic basin must contain a critical value, and the boundary of any rotation domain is contained in the closure of the union of all critical orbits of f.

A transcendental meromorphic function f need not have any critical values, but if we instead consider the set of singular values (the closure of all critical and asymptotic values of f), then the above relationships also hold in this context. However, another type of invariant component may occur for transcendental functions: In an invariant Baker domain, the iterates of the function converge to the singularity at infinity.

In 1993, Bergweiler asked whether there is also a relationship between the singular values and Baker domains. In particular, he asked whether it is possible that a meromorphic function has Baker domains if the forward orbit of all singular values is bounded.

We answer this question in the positive, by showing that there is a transcendental meromorphic function with a Baker domain such that every singular value is a super-attracting periodic point of period 2. Moreover, this Baker domain contains arbitrarily large round annuli of definite modulus, which answers a question raised by Mihaljevic and the speaker in 2013. The construction uses quasiconformal surgery.

Finding polynomial roots using complex analysis, dynamical systems, computer algebra

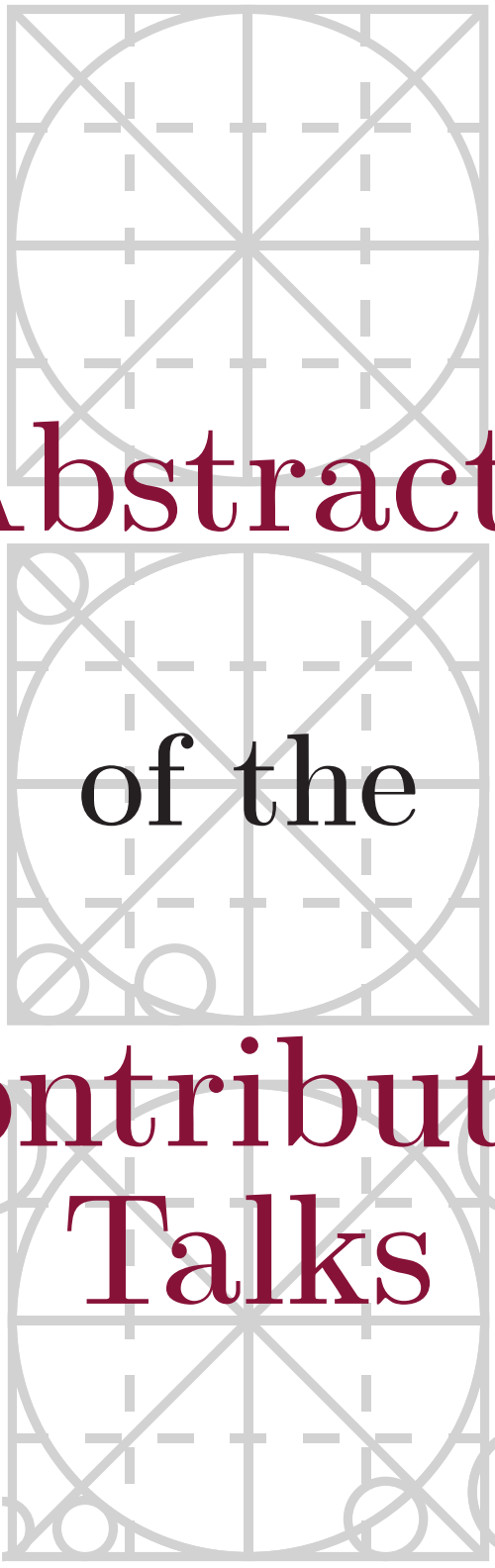
Dierk Schleicher

Institut de Mathématiques de Marseille, France.

One of the classical problems in all areas of mathematics is to find roots of complex polynomials. It is well known that this can be done only by methods of approximation. We discuss three classical methods: the Newton, Weierstrass, and Ehrlich-Aberth methods; these are complex analytic maps that, under iteration, are supposed to converge to one root, resp. all roots of the polynomial. Locally, these methods converge fast, but the global dynamical properties are hard to describe.

We introduce these complex analytic dynamical systems and describe recent progress towards their global dynamical properties. In particular, the Newton and Weierstrass methods are not globally convergent: for open sets of polynomials there are open sets of initial points that fail to converge to roots. Moreover, for Weierstrass and Ehrlich-Aberth, there are orbits that are always defined and converge, but not to roots. For Newton, there is meanwhile a rich theory about its global dynamics, but there are many open questions for all these methods.

Much of this is joint work with members of my ERC team, in particular my PhD student Bernhard Reinke (who will present more details on Thursday), as well as with colleagues.



Abstracts of the Contributed Talks

Transcendental dynamics and iterations on infinite-dimensional Teichmüller spaces

Konstantin Bogdanov

Aix-Marseille Université, France.

In holomorphic dynamics it is usually important to understand and describe the dynamical behavior of critical (or singular) orbits. For quadratic polynomials, this leads to the study of the Mandelbrot set and of its complement. In our talk we present a theorem which classifies within certain families the transcendental entire functions for which all singular values escape, that is, inside of the complement of the "transcendental analogue" of the Mandelbrot set.

A key to the proof of the theorem is a generalization of the celebrated Thurston's Topological Characterization of Rational Functions, but for the case of infinite rather than finite post–singular set. Analogously to the Thurston's theorem one defines the sigma–iteration on the Teichmüller space and investigates the question of convergence. But this time the underlying Teichmüller space is infinite–dimensional which leads to a completely different theory.

An alternative construction of Hausdorff dimension 1 Julia sets

Jack Burkart

(in collaboration with Kirill Lazebnik)

Stony Brook Univeristy, New York, United States of America.

Hausdorff dimension of a Julia set of a transcendental entire function must belong to $[1, 2]$, and all values can be attained. Bishop's example is currently the only known example where the value 1 is attained, and in this talk we show how to give an alternative construction of such a function. Time permitting, we will also discuss some open problems our approach could potentially address.

The parameter space of a family of meromorphic functions

Tao Chen

(in collaboration with Linda Keen)

The City University of New York, New York, United States of America.

The family of $\lambda \tan^p z^q$, where $p \geq 1$, $q \geq 1$ are integers and $pg > 1$ is one dimension slice in the generalize Nevanlinna family. Their dynamics depends on the behavior of one asymptotic value. In this talk, we will describe a parameter space of the family $\lambda \tan^p z^q$.

Internal dynamics and connectivity of multiply connected wandering domains of meromorphic functions

Gustavo Rodrigues Ferreira

Open University, United Kingdom.

In this talk, we discuss how the nine-way classification scheme devised by Benini et al. for the dynamical behaviour in simply connected wandering domains of entire functions, based on the long-term behaviour of the hyperbolic metric and the distance between orbits and the domains' boundary, carries over to the general case of multiply connected wandering domains of meromorphic functions. We see that, although the "convergence to the boundary" classification requires only minor adjustments, the "hyperbolic distance between orbits" classification is drastically overturned. We also find connections between a wandering domain's connectivity and its possible long-term behaviour. Finally, to further illustrate the diversity of possible behaviours in a general setting, we give an example of a wandering domain without a well-defined eventual connectivity.

Interior Dynamics of Fatou Sets

Mi Hu

University of Parma, Italy.

In particular, let f be a holomorphic polynomial of degree $m \geq 2$ on C and $\mathcal{A}(p)$ be a basin of attraction of an attracting fixed point p, we prove that there is a constant C so that for every $z_0 \in \mathcal{A}(p)$, there exists a point q such that $f^n(q) = p$ for some integer n and $d_{\mathcal{A}}(z_0, q) \geq C$, where $d_{\mathcal{A}}$ is the Kobayashi metric on $A(p)$.

Acknowledgments: I am very grateful to Professor John Erik Fornaess for his guidance about this work

Achievable connectivities of Fatou components for a family of rational maps

Dan Paraschiv

Universitat de Barcelona, Spain.

In this talk we study the connectivity of Fatou components for maps in a large family of singular perturbations. We prove that, for some parameters inside the family, the dynamical planes for the corresponding maps present Fatou components of arbitrarily large connectivity and we determine precisely these connectivities. In particular, these results extend the ones obtained inby J. Canela previously.

Entire functions with Cantor bouquet Julia sets

Leticia Pardo Simón
(in collaboration with Lasse Rempe)

Intitute for Mathematics, Polish Acaedmy of Sciences, Poland.

A transcendental entire function with bounded singular set that is hyperbolic and has a unique Fatou component is said to be of *disjoint type*. The Julia set of any disjoint-type function of finite order is known to be a collection of curves that escape to infinity and form a *Cantor bouquet*, i.e., a subset of $\mathbb{C}$ ambiently homeomorphic to a straight brush. We show that there exists f of disjoint type whose Julia set $J(f)$ is a collection of escaping curves, but $J(f)$ is not a Cantor bouquet. On the other hand, we prove that if f of disjoint type and $J(f)$ contains an *absorbing Cantor bouquet*, that is, a Cantor bouquet to which all escaping points are eventually mapped, then $J(f)$ must be a Cantor bouquet itself.

The Weierstrass root finder is not generally convergent

Bernhard Reinke

(in collaboration with Dierk Schleicher and Michael Stoll)

Aix-Marseille University, France.

The Weierstrass root finder is not generally convergent

Finding roots of univariate polynomials is one of the fundamental tasks of numerics, and there is still a wide gap between root finders that are well understood in theory and those that perform well in practice.

We will give an overview of root-finding methods and their interpretation as complex dynamical systems. The main focus will be the Weierstrass/Durand–Kerner method and its similarities and differences to the Newton and the Ehrlich–Aberth methods.

In particular, we show how to use methods from computer algebra to investigate (and/or establish) the existence of attracting periodic cycles, as well as diverging orbits, and present explicit examples of both phenomena for the Weierstrass method. We thereby settle the conjecture by Smale that the Weierstrass root finder is not generally convergent.

Comparison of Newton and Ehrlich-Aberth rootfinding dynamical systems: practice and theory

Sergey Shemyakov

Aix-Marseille University, France.

Newton's and Ehrlich-Aberth's are two dynamical systems naturally arising from the problem of finding roots of holomorphic functions. In my talk I speak about the published experimental paper which compares the efficiency of two methods for different polynomials. I explain some known theoretical and practical results on Newton's method and Ehrlich-Aberth's method and present a practical observation of when Newton's is faster. In the end I give a theoretical idea trying to explain the experimental results.

Dynamics of Zorich maps

Athanasios Tsantaris

University of Nottingham, United Kingdom.

In the theory of one dimensional holomorphic dynamics, one of the most well studied families of maps is the exponential family $E_\lambda(z) := \lambda e^z$. Zorich maps are the quasiregular higher dimensional analogues of the exponential map on the plane. For the exponential family $E_\lambda(z) := \lambda e^z$, $\lambda > 0$ it is generally well known that for $0 < \lambda \leq 1/e$ the Julia set of E_λ is a "Cantor Bouquet" while for $\lambda > 1/e$ the Julia set is the entire complex plane. In this talk we will discuss how this dichotomy and many other facts about the exponential family generalize to the higher dimensional setting of Zorich maps.

Docile entire functions

James Waterman
(in collaboration with Lasse Rempe)

University of Liverpool, United Kingdom.

Several important problems in complex dynamics are centered around the local connectivity of Julia sets of polynomials and of the Mandelbrot set. Importantly, when the Julia set of a polynomial is locally connected, the topological dynamics of the map can be completely described as a quotient of a power map on the circle.

Local connectivity of the Julia set is less significant for transcendental entire functions. Nevertheless, by restricting to a class of transcendental entire functions, known as docile functions, we obtain a similar concept by describing the topological dynamics as a quotient of a simpler disjoint-type map. We will discuss the notion of docile functions, as well as some of their properties.

A class of Newton maps with Julia sets of Lebesgue measure zero

Mareike Wolff

University of Kiel, Kiel, Germany.

In 1987, McMullen [2] proved that for any entire function whose postsingular set is a compact subset of the Fatou set, if the density of the Julia set in all sufficiently large disks is bounded away from one, then the Lebesgue measure of the Julia set is zero. This was generalised by Stallard [3], Jankowski [4] and Zheng [5] to meromorphic functions whose possibly unbounded postsingular set has, with certain exceptions within a bounded subset of the complex plane, positive distance to the Julia set. We are able to extend these results to functions with unbounded sequences of postsingular values whose distance to the Julia set tends to zero, provided that the density of the Julia set close to these postsingular values is bounded away from one.

As an application, consider the function f arising from Newton's method for $g(x) = \int_0^z p(t)\exp(q(t))dt + c$ where p and q are polynomials. The set of critical points of f consists of the simple zeros of g, which are superattracting fixed points of f, and the zeros $z_1, \ldots, z_N$ of g'' which are not zeros of g or g'. If $f^n(z_j)$ is attracted by a periodic cycle for all $j \in \{1, \ldots, N\}$, then we are able to show that the Julia set of f has Lebesgue measure zero. Toguerther with a theorem by Bergweiler [1], this implies that $f^n(z)$ converges to zeros of g almost everywhere in the complex plane if this is the case for each z_j.

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On the formulas of meromorphic functions with periodic Herman rings

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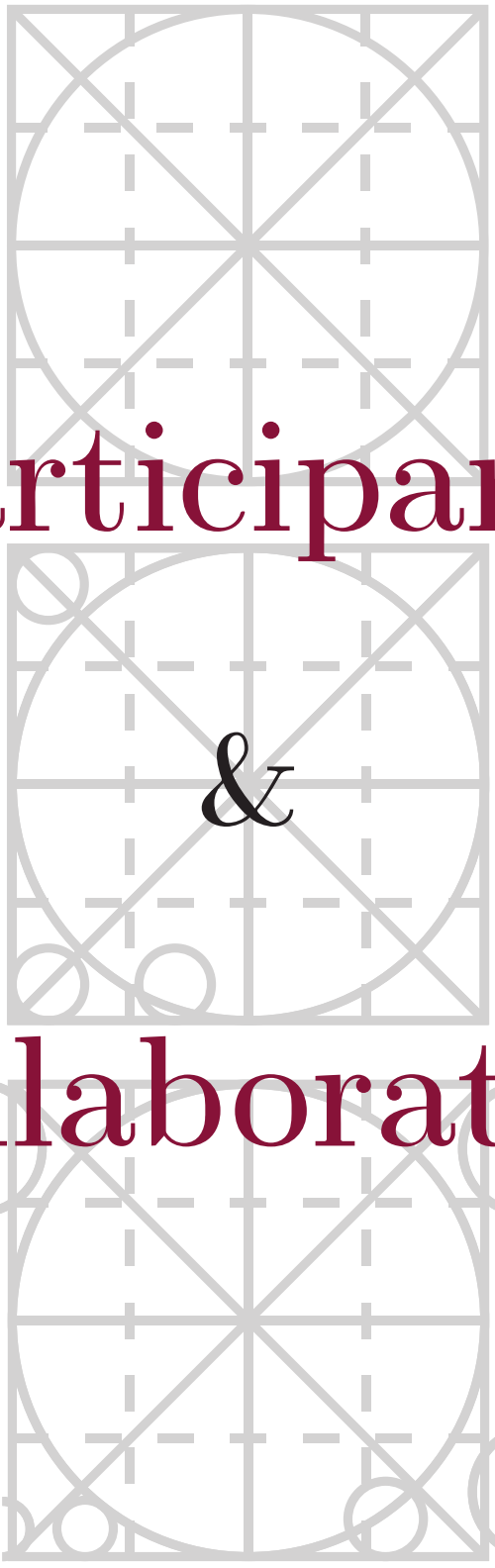
We construct some explicit formulas of rational maps and transcendental meromorphic functions having Herman rings of period strictly larger than one.

This gives an answer to a question raised by Shishikura in the 1980s. Moreover, some formulas of rational maps with nested Herman rings are also found.

To obtain the formulas of transcendental meromorphic functions having periodic Herman rings, we first find an explicit family of transcendental entire functions having Siegel disks of all possible periods and rotation numbers. This is based on proving the existence of a Mandelbrot-like set of period one in the corresponding parameter space.



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