A time-dependent energy-momentum method

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Roughly speaking, the Marsden-Weinstein theorem allows for the 'simplification' of a Hamiltonian system on a symplectic manifold that admits a Hamiltonian Lie group of symmetries of a certain type to new Hamiltonian systems, the so-called reduced Hamiltonian systems, on the quotients of submanifolds that are invariant relative to the evolution of the original Hamiltonian.

In my talk, I will present a time-dependent generalisation of the so-called energy-momentum method, originally designed for studying the stability of equilibrium points of a reduced, autonomous, Hamiltonian system on a symplectic manifold. The energy-momentum method has proven to be very fruitful and many applications and generalisations have been accomplished over the years.

The detailed structure of my talk goes as follows. I shall first introduce some fundamental notions from symplectic geometry and Lyapunov stability theories such as momentum maps, the Marsden-Weinstein theorem, and stability theorems, extending classical results to a time-dependent setting on manifolds. Next, I will define a notion of relative equilibrium points that are points that project onto equilibrium points after the Marsden-Weinstein reduction to a quotient space. Remarkably, it may happen that an equilibrium point of a reduced Hamiltonian system is not the projection of an equilibrium point of the initial one. Then, I shall comment on the properties of these equilibrium points and their relation to the behavior of the initial Hamiltonian system at relative equilibrium points. Finally, I will introduce some conditions determining the stability of equilibrium points of reduced Hamiltonian systems and several other related results. If time permits, I will shortly discuss the use of not Ad-equivariant momentum maps in the previous formalism.