Trading and Hedging Bitcoin Volatility

Santander Financial Engineering School

Carol Alexander

Professor of Finance, University of Sussex
Visiting Professor, HSBC Business School, Peking University

18 March 2021

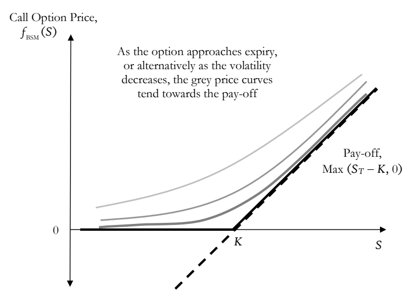
1. Options Fundamentals

- A standard European option is like a 'ticket' that gives the holder the right to buy (call option) or sell (put option) an 'underlying' financial instrument with market price S_t at time t
  - at a fixed price called the strike price, K
  - on the maturity date, T years after its issue
- Running time from issue of the option (time 0) until the option expires is denoted t, so $T-t$ denotes the time to expiry in years
- If the option is held to expiry (but few are), the pay-off is
  - $\max\{S_T - K, 0\}$ for a standard call option
  - $\max\{K - S_T, 0\}$ for a standard put option

Call Option Prices

Strike	Maturity (days)											
	40	50	60	70	80	90	100	110	120	130	140	150
80	20.372	20.495	20.611	20.690	20.800	20.916	20.972	21.063	21.166	21.220	21.329	21.408
82	18.383	18.510	18.632	18.716	18.832	18.956	19.019	19.118	19.228	19.291	19.409	19.498
84	16.396	16.528	16.660	16.751	16.876	17.010	17.081	17.191	17.311	17.383	17.511	17.611
86	14.414	14.557	14.700	14.803	14.937	15.085	15.170	15.290	15.425	15.506	15.645	15.758
88	12.445	12.602	12.760	12.882	13.029	13.195	13.299	13.431	13.580	13.675	13.827	13.957
90	10.501	10.681	10.860	11.010	11.178	11.362	11.490	11.634	11.801	11.911	12.077	12.226
92	8.605	8.818	9.030	9.206	9.404	9.605	9.760	9.924	10.107	10.230	10.413	10.580
94	6.794	7.051	7.303	7.508	7.729	7.953	8.140	8.319	8.520	8.659	8.852	9.032
96	5.128	5.426	5.720	5.950	6.194	6.441	6.656	6.847	7.059	7.209	7.411	7.604
98	3.666	4.001	4.328	4.576	4.836	5.100	5.325	5.529	5.739	5.904	6.114	6.309
100	2.473	2.820	3.151	3.415	3.676	3.945	4.171	4.384	4.585	4.758	4.970	5.171
102	1.577	1.898	2.215	2.479	2.721	2.981	3.202	3.413	3.609	3.780	3.985	4.186
104	0.960	1.228	1.502	1.743	1.969	2.208	2.414	2.615	2.797	2.963	3.145	3.345
106	0.560	0.768	0.993	1.194	1.396	1.612	1.793	1.974	2.137	2.292	2.456	2.645
108	0.324	0.469	0.645	0.806	0.969	1.157	1.317	1.475	1.620	1.753	1.899	2.079
110	0.185	0.285	0.410	0.535	0.662	0.823	0.958	1.096	1.222	1.337	1.461	1.624
112	0.103	0.171	0.260	0.351	0.449	0.580	0.697	0.810	0.919	1.019	1.119	1.260
114	0.057	0.102	0.166	0.232	0.306	0.408	0.506	0.598	0.688	0.771	0.856	0.976
116	0.032	0.061	0.108	0.153	0.208	0.289	0.370	0.443	0.511	0.581	0.654	0.756
118	0.018	0.039	0.071	0.100	0.142	0.204	0.271	0.330	0.381	0.439	0.501	0.584
120	0.010	0.024	0.047	0.064	0.097	0.145	0.201	0.246	0.287	0.334	0.383	0.451

Option Price vs Underlying Price



Moneyness

- The moneyness of an option is a measure of it's worth if it were exercised now
- Exact definitions of moneyness differ by author – e.g. some use an approximate definition $S_t - K$, others discount these quantities and yet others use a (discounted) ratio of S_t and K
- When moneyness is defined as a ratio we distinguish moneyness > 1, 1, < 1 according as:
  - Calls are in-the-money (ITM), at-the-money (ATM), or out-of-the-money (OTM)
  - Puts are out-of-the-money (OTM), at-the-money (ATM) or in-the-money (ITM)

Market Prices vs Model Prices

- Options may be traded on the secondary market at any time t between the time of issue and the maturity date T
- Like any other traded asset, in a liquid market their market price is set by supply and demand
- The model (or fair) price of an option depends on the underlying asset's volatility – unlike the price of a futures
- Hence, the fair price of an option depends on the model we choose for the stochastic process for the underlying
- Parameters of an option pricing model are (usually) calibrated by equating the market and model prices for vanilla options

Fair Price of a Futures Contract

- Denote by S_t the spot market price of a risky asset time t. We assume the asset is (a) tradable both long and short, and (b) may pay dividends or have carry costs
- $F_{t,T}$ is the time t forward price of this asset for exchange at some future maturity T
- Standard 'no-arbitrage' arguments show that there is a unique theoretical or model (or 'fair') price

$$F_{t,T} = e^{(r-y)(T-t)} S_t$$

where r denotes the risk-free rate of return (also called the discount rate) and y denotes the dividend yield/carry cost – both r and y are assumed constant throughout this course

Fair Prices for Options

- A form of 'no arbitrage' may also be used to price options, but now the model price is not unique because the price of an option depends on the volatility of S_t so different volatility models give different option prices
- In a complete market we can add other traded assets or instruments to an option so that the resulting portfolio is risk free – in other words the option can be perfectly hedged. So the portfolio should return the risk-free rate
- Since it has no risk, this risk-free portfolio will have the same value today for all investors regardless of their attitude to risk
- And because the hedging instruments have market prices, which are the same for all investors, the option must also have the same value for all investors. So we use the value we derive for a risk-neutral investor

Risk-Neutral Measures

- A risk-neutral investor's beliefs about a risky asset price are captured by a risk-neutral measure i.e. a measure under which the the expected total return on any risky asset is the risk-free rate r
- Harrison and Kreps (1984) $\Rightarrow$ in a complete, no-arbitrage market there is a unique risk neutral measure, $\mathbb{Q}$
- Sometimes holding the underlying pays a dividend with yield $y > 0$ (e.g. a stock)
- Sometimes it incurs a carry cost so $y < 0$ (e.g. a commodity)
- A price process S_t for the risky asset is measured without dividends/costs so its expected return under a risk-neutral measure will be $r - y$.

Option Price Derivation

- In a complete, no-arbitrage market the price of any option may be derived using the principle of risk-neutral valuation (RNV):

$$\text{RN Option Price at time } t = e^{-r(T-t)} \mathbb{E}_t^{\mathbb{Q}} [\text{Pay-off}]$$

- The model price of the option uses the pricing measure $\mathbb{M}$ induced by the stochastic volatility model

$$\text{Model Option Price at time } t = e^{-r(T-t)} \mathbb{E}_t^{\mathbb{M}} [\text{Pay-off}]$$

- Popular volatility models yield measures $\mathbb{M}$ where the above expectation for a vanilla option may be written in closed-form

Review of the Black-Scholes Model

- The Black-Scholes (BS) model assumes that the underlying S_t follows a GBM:

$$\frac{dS_t}{S_t} = (r - y)dt + \sigma dW_t$$

- The price process measure for log returns is normal, i.e. as seen from time 0:

$$\ln(S_t) - \ln(S_0) \sim N\left(\left(r - y - \tfrac{1}{2}\sigma^2\right)t, \sigma^2 t\right)$$

- Equivalently the measure for the price S_t is a lognormal density:

$$S_t \sim \ln N\left(\ln S_0 + \left(r - y - \tfrac{1}{2}\sigma^2\right)t, \sigma^2 t\right)$$

Black-Scholes(-Merton) Formula

Call option price:

$$C(K,T) = e^{-y(T-t)} S_t \Phi(d_{1t}) - e^{-r(T-t)} K \Phi(d_{2t})$$

Put option price:

$$P(K,T) = -e^{-y(T-t)} S_t \Phi(-d_{1t}) + e^{-r(T-t)} K \Phi(-d_{2t})$$

where Φ denotes the standard normal distribution function and $C(K,T)$ and $P(K,T)$ are the BS price of a standard European call and put option

The variables d_1 and d_2 reflect the log moneyness of the option......

Moneyness

- We define the moneyness of an option as

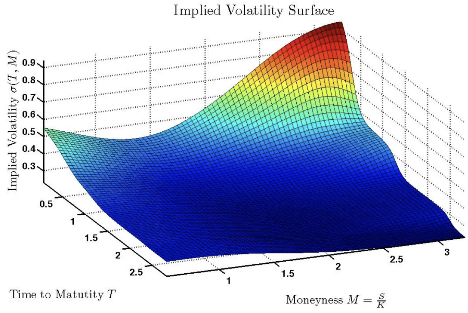
$$M_t = \frac{S_t e^{-y(T-t)}}{K e^{-r(T-t)}}$$

and M_t is > 1, $= 1$ or < 1 depending on whether a call/put option is ITM/OTM, ATM or OTM/ITM

- The log moneyness is $m_t = \ln M_t$
- For calls, m_t is > 0 for ITM, 0 for ATM, < 0 for OTM (and conversely for puts)
- And d_1 and d_2 may be written

$$d_{1t} = \frac{m_t}{\sigma\sqrt{T-t}} + \frac{\sigma\sqrt{T-t}}{2}, \quad d_{2t} = d_{1t} - \sigma\sqrt{T-t}$$

2. Implied Volatility Smile Surfaces



Transformations of Option Prices

- Consider a set of market (or model) prices $f(K, T|S, \sigma)$ for vanilla options of various strikes and maturities, all on the same underlying asset with spot price S and price process volatility σ
- Each such set of smoothed prices may be transformed into:
  - An implied volatility surface
  - A set of implied densities for S
  - A local volatility surface
- For simplicity of notation, in this section we assume there are no dividends and the discount rate r is zero

Market Implied Volatility

- To obtain a BS option price, one needs to forecast an average value σ for volatility over the life of the option. The other inputs to the BS model (current underlying price S_t, risk-free rate r, dividend yield y and the parameters T and K) are observable
- But in practice, we actually observe the market price for the option and not the average volatility
- The option's market implied volatility is the σ that equates the BS price with the observed market price of the option
- It is the average volatility of the underlying, over the life of the option that is implicit in the market price of the option, according to the BS model

Market vs Model Implied Volatility

- The market (model) implied volatility $\theta(K, T|S)$ of a vanilla option is the constant volatility which equates the BS price of the option to its market (model) price
- By put-call parity, a call and a put of the same strike and maturity should have identical market implied volatilities
- A set of vanilla option prices with different strikes and maturities yields an implied volatility surface
- Standard features include a negative volatility skew and a convergent volatility term structure

How to Compute the Market Implied Volatility

- Denote the market price of a vanilla European option with strike K and maturity T by $f^m(K, T)$
- Denote the market implied volatility of the option by $\theta(K, T)$
- To find $\theta(K, T)$ we use a numerical method to back-out the implied volatility from an equation which sets the market price of the option equal to its BS model price:

$$f^m(K, T) = f^{BS}(\theta(K, T))$$

- E.g. set as objective the difference between the model price and the market price of the option and use the Excel solver to set the objective value to zero

Interpolation Methods

- Strike dimension typically represented by moneyness
  - Over a long period, range of strikes varies considerably but range of moneyness is fairly constant
  - Latest methods favour piecewise cubic Hermite splines because of their shape preserving properties
- Maturity dimension:
  - Interpolation usually done over implied variance
  - Typically linear but could use a parametric form
- Resulting surface must be checked for no-arbitrage constraints

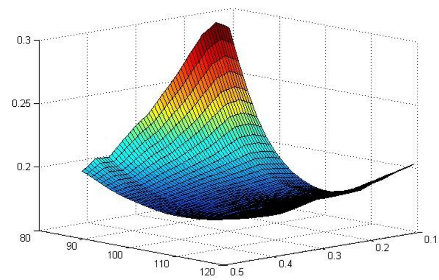
Implied Volatility Smiles

- We almost always find that we cannot replicate the market prices for all options on the same underlying using a single, constant volatility in the BS formula
- That is, the market implied volatility depends on the strike and maturity of the option
- The smile effect refers to the empirical fact that for most underlying assets a plot of implied volatility against strike (or moneyness) has a smile shape
- The smile often has a strong negative skew especially in equity markets

Call Option Prices

Strike	Maturity (days)											
	40	50	60	70	80	90	100	110	120	130	140	150
80	20.372	20.495	20.611	20.690	20.800	20.916	20.972	21.063	21.166	21.220	21.329	21.408
82	18.383	18.510	18.632	18.716	18.832	18.956	19.019	19.118	19.228	19.291	19.409	19.498
84	16.396	16.528	16.660	16.751	16.876	17.010	17.081	17.191	17.311	17.383	17.511	17.611
86	14.414	14.557	14.700	14.803	14.937	15.085	15.170	15.290	15.425	15.506	15.645	15.758
88	12.445	12.602	12.760	12.882	13.029	13.195	13.299	13.431	13.580	13.675	13.827	13.957
90	10.501	10.681	10.860	11.010	11.178	11.362	11.490	11.634	11.801	11.911	12.077	12.226
92	8.605	8.818	9.030	9.206	9.404	9.605	9.760	9.924	10.107	10.230	10.413	10.580
94	6.794	7.051	7.303	7.508	7.729	7.953	8.140	8.319	8.520	8.659	8.852	9.032
96	5.128	5.426	5.720	5.950	6.194	6.441	6.656	6.847	7.059	7.209	7.411	7.604
98	3.666	4.001	4.328	4.576	4.836	5.100	5.325	5.529	5.739	5.904	6.114	6.309
100	2.473	2.820	3.151	3.415	3.676	3.945	4.171	4.384	4.585	4.758	4.970	5.171
102	1.577	1.898	2.215	2.479	2.721	2.981	3.202	3.413	3.609	3.780	3.985	4.186
104	0.960	1.228	1.502	1.743	1.969	2.208	2.414	2.615	2.797	2.963	3.145	3.345
106	0.560	0.768	0.993	1.194	1.396	1.612	1.793	1.974	2.137	2.292	2.456	2.645
108	0.324	0.469	0.645	0.806	0.969	1.157	1.317	1.475	1.620	1.753	1.899	2.079
110	0.185	0.285	0.410	0.535	0.662	0.823	0.958	1.096	1.222	1.337	1.461	1.624
112	0.103	0.171	0.260	0.351	0.449	0.580	0.697	0.810	0.919	1.019	1.119	1.260
114	0.057	0.102	0.166	0.232	0.306	0.408	0.506	0.598	0.688	0.771	0.856	0.976
116	0.032	0.061	0.108	0.153	0.208	0.289	0.370	0.443	0.511	0.581	0.654	0.756
118	0.018	0.039	0.071	0.100	0.142	0.204	0.271	0.330	0.381	0.439	0.501	0.584
120	0.010	0.024	0.047	0.064	0.097	0.145	0.201	0.246	0.287	0.334	0.383	0.451

Implied Volatility Surface



Why the Smile?

- If traders believed BS assumptions, market prices $\equiv$ BS model prices
- i.e. BS implied volatility would be same for all options on the same underlying asset
- So the market does not believe in the assumptions of the BS model. Participants believe log returns are heavy-tailed
- An OTM option has more chance to become ITM than the BS model predicts, so its market price > BS model price
- The BS price can only match the market price by increasing the volatility – this is the only parameter that the BS model is free to change, hence:

  OTM implied volatility > ATM implied volatility

Why the Skew?

- In equity markets a large price fall is bad news for the shareholders, and tends to be followed by further price falls, whereas a price rise is good
- So, in turbulent periods, equity returns have negatively skewed distributions
- Market makers are able to charge a particularly high price for an OTM put (higher than the corresponding OTM call) because there is a high demand for the insurance offered by these puts from risk-averse investors
- This highly-priced, limited supply of OTM puts will be met by heavy demand as long as investors hold pessimistic views about the possibility of a price crash

3. Trading Volatility

- An option is a trade on both the price of the underlying and the implied volatility
- An ATM straddle attempts to isolate the volatility component so that the trade is a pure volatility trade
- But as soon as the underlying price moves it is no longer ATM, and the straddles becomes a directional trade
- Thus, options strategies for trading volatility require frequent rebalancing. Unfortunately, options are also relatively expensive to trade
- The innovation of variance swaps is because they, and their derivative products, are relatively inexpensive pure volatility trades

Why Trade Volatility

Reasons to trade volatility as an asset class in its own right:

- Diversification: e.g. equity volatility is highly negatively correlated with the equity price
- Hedging: e.g. trading forward volatility provides a vega hedge for forward start options
- Arbitrage: e.g. taking opposite positions on volatilities of different underlyings
- Speculation: e.g. a pure bet on realised volatility

Variance Risk Premium

- The variance risk premium (VRP) is the reward required by a risk-averse investor to compensate for the risk of both stochastic volatility and jumps in price of the underlying asset
- Every stochastic volatility-jump model has parameters which are calibrated in the risk-neutral measure by equating the model price of a standard European option to its market price
- We may also estimate the model parameters in the physical measure by applying advanced econometric techniques (e.g. particle filter) to time series data on the underlying
- Then attribute the VRP to different features of volatility e.g. mean-reversion speed or long-term volatility by taking the difference between the two parameter estimates

Model-Free Approach

- An alternative 'model-free' approach to measuring the VRP equates it with the expected P&L on a variance swap
- A variance swap is an OTC instrument which exchanges:
  - A realised variance (RV), measured over the life of the swap
  - A fixed variance swap rate (VSR)
- Market VSRs are set by supply and demand
- Model VSRs are based on a 'model-free' formula which determines the VSR as an integral of vanilla option prices
- This approach is called 'model free' because the (approximate) fair-value VSR doesn't depend on the specification of the model for the underlying price and volatility processes – much!

Variance Swaps

- A variance swap of maturity T is an OTC instrument that exchanges realised variance, RV_T (typically as the sum of squared daily log returns over the life of the swap) with a fixed swap rate, K_T, with pay-off $[RV_T - K_T]$ per $ per % point
- Fair-value swap rate K_T^* is fixed so that $\mathbb{E}_0^{\mathbb{Q}}\left[RV_T - K_T^*\right] = 0$
- Market swap rates K_T can differ considerably from fair-value swap rate K_T^*, especially during excessively volatile markets
- The variance risk premium of maturity T can be defined as the pay-off to a long position on a variance swap of maturity T
- Usually the variance risk premium is small and negative, but before a period of unexpectedly high volatility it becomes very high and positive $\Rightarrow$ writing variance swaps is a very risky business

Example

Suppose the 30-day variance swap rate is 18% and the expected 30-day realized variance is 20%, both quoted as annual volatilities. What is the expected pay-off to a long position of $100 per basis point on this swap?

The point value of $100 per basis point is the same as $10,000 per percentage point. Hence the expected pay-off is

$$(20^2 - 18^2) \times \$10,000 = \$760,000$$

Remarks: The realised variance (typically defined as the sum of squared log returns over the life of the swap) can only be calculated ex-ante, and the T&C of the swap will specify the day-count convention for annualisation.

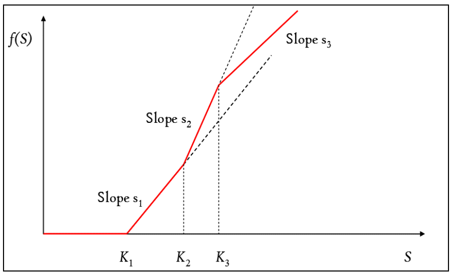
Fair-Value Swap Rates

- The log contract of maturity T has pay-off $\ln(S_T/S_0)$ where S_0 is the underlying price today
- Neuberger (1994): If the underlying price follows a GBM then the expected pay-off to a short position on the log contract of maturity T is the continuously-monitored realised variance

$$T^{-1}\int_0^T \sigma^2 dt$$

- The fair-value swap rate K^* is the risk-neutral expectation of the discretely-monitored realised variance
- So an approximate formula for K^* is obtained by replicating the pay-off to the log contract

Gamma Replication



Buy s_1 calls at strike K_1, $s_2 - s_1$ calls at strike K_2, $s_3 - s_2$ calls at strike K_3, ...

Replication of the Short Log Contract

- Replication of a pay-off profile $g(S,T)$ may be achieved using a *gamma weighted* portfolio of vanilla calls and puts on S of the same maturity as the pay-off (Carr and Madan, 2001)
- That is, the time 0 value of a replication portfolio is

$$\sum_{i=1}^{n} \Delta k_i g_{SS}|_{S=k_i} f(k_i,T)$$

- The gamma of the short log contract is S^{-2}, and we set

$$\Delta k_i = {}^1\!/_2\,(k_{i+1} - k_{i-1})$$

- So the replication portfolio for a short log contract has value

$$V_T = {}^1\!/_2 \sum_i (k_{i+1} - k_{i-1})\, k_i^{-2} f(k_i, T)$$

Derivation of the Volatility Index Formula

Use Ito's lemma to write the GBM as:

$$d\ln S_t = \left(r - {}^1\!/_2\,\sigma^2\right) dt + \sigma dW_t$$

Integrate:

$$\ln(S_T/S_0) = rT - {}^1\!/_2 \int_0^T \sigma^2 dt + \int_0^T \sigma dW_t$$

That is,

$$T^{-1}\int_0^T \sigma^2 dt = 2r - 2T^{-1}\ln(S_T/S_0) + 2\int_0^T \sigma dW_t$$

Now take expectations under the risk-neutral measure and use $\mathbb{E}^{\mathbb{Q}}\left[\int_0^T \sigma dW_t\right] = 0$ .....

Fair Value for a Continuously-Monitored Variance Swap

$$K_T^* \approx \mathbb{E}^{\mathbb{Q}}\left[T^{-1}\int_0^T \sigma^2 dt\right] = 2r - 2T^{-1}\mathbb{E}^{\mathbb{Q}}\left[\ln(S_T/S_0)\right]$$
$$= 2r + 2T^{-1}V_T$$

Thus, assuming $r = 0$ the variance swap rate is approximated as:

$$K_T^* \approx T^{-1}\sum_i (k_{i+1} - k_{i-1})\, k_i^{-2} f(k_i, T)$$

Alternatively

$$K_T^* \approx 2T^{-1}\sum_i \Delta k_i\, k_i^{-2} f(k_i, T)$$

We use OTM options (low strike puts and high strike calls) because their prices are more liquid

The VIX and Other Volatility Indices

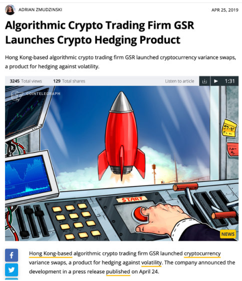
The VIX index is the (annualised) volatility of the 30-day fair-value variance swap rate on the S&P 500 index

It is nick-named the 'investor's fear gauge' because it represents the implied volatility smile on the main large-cap US stock index

Similar 30-day fair-value variance swap rate indices are quoted on most major stock indices

- VFTSE
- VDAX
- VSTOXX
- VCAC, etc....

2. Bitcoin Options and Variance Swaps


ADRIAN ZMUDZINSKI APR 25, 2019

Algorithmic Crypto Trading Firm GSR Launches Crypto Hedging Product

Hong Kong-based algorithmic crypto trading firm GSR launched cryptocurrency variance swaps, a product for hedging against volatility.

3245 Total views 129 Total shares Listen to article 1:31

Hong Kong-based algorithmic crypto trading firm GSR launched cryptocurrency variance swaps, a product for hedging against volatility. The company announced the development in a press release published on April 24.

Alexander, C. and A. Imeraj (2020) The Bitcoin VIX and its Variance Risk Premium. *Journal of Alternative Investments* DOI: 10.3905/jai.2020.1.112

Alexander, C. and A. Imeraj (2020) BVIN: The Bitcoin Implied Volatility Index. CryptoCompare

Example – Deribit Option Prices in April 2020

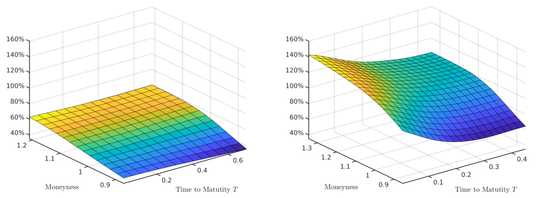
T_1-Strike	Type	Price in $
4250	P	8.967
4375	P	10.25
4500	P	20.5
4625	P	32.03
4750	P	49.97
4875	P	76.86
5000	P	115.31
5125	C	180.66
5250	C	130.69
5375	C	93.53
5500	C	69.19
5625	C	44.84
5750	C	30.75
5875	C	23.06
6000	C	15.375
6125	C	10.25
6250	C	8.97
6375	C	7.69
6500	C	6.41
6625	C	6.41

T_2-Strike	Type	Price in $
4000	P	16.66
4125	P	21.78
4250	P	33.31
4375	P	48.69
4500	P	67.91
4625	P	94.81
4750	P	126.84
4875	P	169.13
5000	P	220.375
5125	P	285.72
5250	C	240.86
5375	C	193.47
5500	C	156.31
5625	C	124.28
5750	C	98.66
5875	C	78.16
6000	C	58.94
6125	C	48.68
6250	C	38.44
6375	C	30.75
6500	C	23.06
6625	C	20.5
6750	C	16.66

Deribit

Implied Volatility Surfaces

Implied volatilities from out-of-the-money options of different strikes and maturities on 15 April 2019 at 02:00:00 UTC (left) and 16 June at 13:00:00 UTC (right). Surfaces interpolated using shape-preserving splines.



Alexander and Imeraj (2020)

Implied VIX from traded options:

$$K_T^* \approx 2T^{-1} \sum_i \Delta k_i \, k_i^{-2} f\left(k_i, T\right)$$

Interpolation to constant maturity gives fair-value variance swap rate:

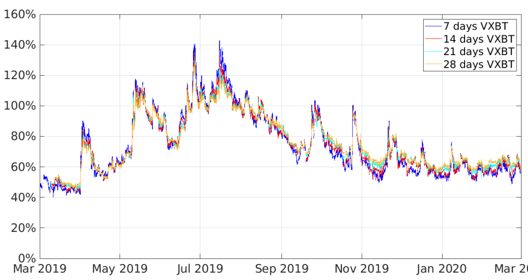
$$\text{VXBT}_T = \sqrt{\omega T_1 K_{T_1}^* + (1-\omega) T_2 K_{T_2}^*}$$

with

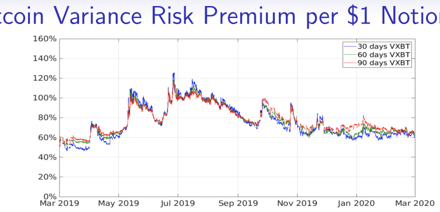
$$\omega = \frac{n_2 - n}{n_2 - n_1} N$$

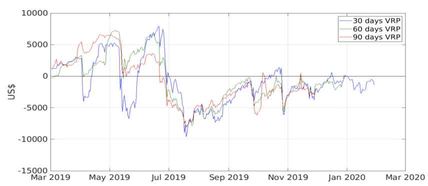
$f(K_i, T)$ is the price of OTM option with maturity T and strike K_i
F_T is the price of the futures contract with maturity T
K_0 is separation strike, i.e. puts for strikes $K_i < K_0$ and calls for strikes $K_i > K_0$
N is the number of seconds in a year
n_i is the number of seconds until maturity T_i, for $i = 1, 2$
n is the number of seconds until maturity T

Variance Swap Rate Term Structure (Short End)



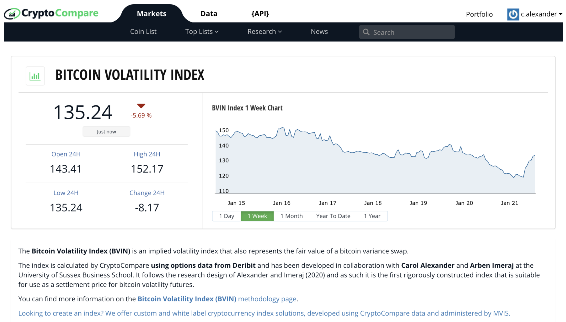
Bitcoin Variance Risk Premium per $1 Notional





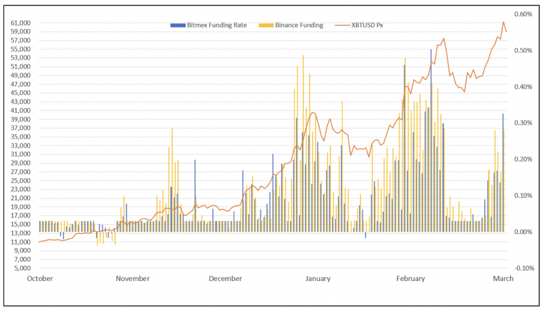
Reverse Leverage Effect in Bubble Regime

CryptoCompare Live-Streamed BVIN Index



BVIN

5. Optimal Hedging with Perpetuals



Alexander, Deng and Zou (2020)

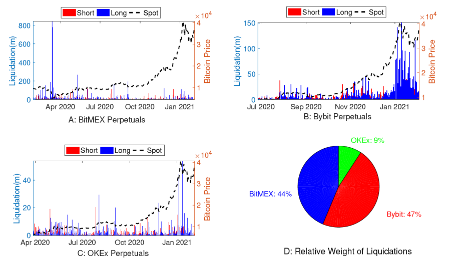
- Perpetuals have no expiry date
- Spot and futures converge via funding rate mechanism
- A positive (negative) rate $\Rightarrow$ P2P payment long $\to$ ($\leftarrow$) short
- Fluctuates $+$ve/$-$ve in contango/backwardation of futures curve
- e.g. funding rate of -0.05% implies payment of \$50 on \$100,000 notional short $\to$ long
- Perpetuals are ideal hedging instruments for miners
- But default risk cannot be ignored due to ultra-high leverage

Margin Mechanism

Settlement	Settlements take place every day at 8:00 UTC. Realized and unrealized session profits (profits made between settlements) are always added in real-time to the equity. However, they are only available for withdrawal after the daily settlement. At the settlement, session profits/losses will be booked to the BTC cash balance.
Contract Size	10 USD
Initial Margin	The initial margin starts with 1.0% (100x leverage trading) and linearly increases by 0.5% per 100 BTC increase in the position size. Initial margin = 1% + (Position Size in BTC) * 0.005%
Maintenance Margin	The maintenance margin starts with 0.525% and linearly increases by 0.5% per 100 BTC increase in the position size. When the account's margin balance is lower than the maintenance margin, positions in the account will be incrementally reduced in order to keep the maintenance margin lower than the equity in the account. Maintenance margin requirements can be changed without prior notice if market circumstances demand such action. Maintenance Margin= 0.525% + (Position Size in BTC) * 0.005%
Mark Price	The mark price is the price at which the perpetual contract will be valued during the trading hours. This can (temporarily) vary from the actual perpetual market price in order to protect market participants against manipulative trading. Mark Price = Index price + 30 seconds EMA of (Perpetual Market Price - Index Price) Where market price is the last traded futures price if it falls between the current best bid and best ask. Otherwise, the market price will be the best bid. If the last traded price is lower than the best bid, or market price will be the best ask, if the last traded price is higher than the best ask.
Delivery/Expiration	No Delivery / Expiration

- Leverage on perpetuals is exceedingly high – around 100X
- If trading losses exceed the maintenance margin, most exchanges employ a platform that automatically and incrementally liquidates the position to cover the margin call

Figure 1: Short and Long Forced Liquidations of bitcoin Perpetual Futures



Note. We plot both the short (in red) and long (in blue) forced liquidations of bitcoin perpetual futures on BitMEX in Panel A (from January 2020 to February 2021), Bybit in Panel B (from June 2020 to Feb. 2021), and OKEx in Panel C (from March 2020 to February 2021). In Panels A-C, the left y-axis is the size (volume) of the forced liquidations in million (m) USD, while the right y-axis is the bitcoin spot price in USD, with the black dashed line denoting the spot price. The pie chart in Panel D plots the relative weight of forced liquidations among the three perpetual futures during the overlapping period (from June 2020 to February 2021). We obtain data from coinalyze.net.

Probability of BitMEX Margin Call

Table 1: Historical Probability of Margin Call for BitMEX Perpetual Futures

	Leverage	8h	1d	2d	15d
Short	5X	0.01%	0.17%	0.63%	21.68%
	20X	5.08%	16.84%	29.46%	70.72%
	50X	25.87%	47.21%	61.41%	88.17%
	100X	48.13%	69.02%	79.11%	93.41%
Long	5X	0.22%	1.29%	3.29%	22.27%
	20X	7.54%	18.66%	29.02%	62.11%
	50X	25.87%	44.64%	56.72%	80.88%
	100X	46.33%	64.76%	74.01%	88.47%

Note. Here we suppose the trader opens a long or short position at the inception and holds it over the hedge horizon. We define a margin call as an event when losses from trading futures exceed the initial margin over the hedge horizon. We use the 1-min historical data of bitcoin perpetual futures on BitMEX from January 1, 2017 to January 31, 2021 to compute the probability of margin calls. We perform computations under different leverage levels, ranging from 5X to 100X, and different hedge horizons, ranging from 8h(hour) to 15d(day).

Basic Problem Formulation

Suppose the platform monitors the margin at discrete intervals Δt

The hedger holds 1 bitcoin at time t and shorts θ units of the futures to hedge spot price volatility until hedge horizon $t + N\Delta t$

Suppose the direct and inverse perpetual futures prices are F_t and $\widehat{F}_t = F_t^{-1}$

Set $\Delta_n S_t := S_{t+n\Delta t} - S_t$ (in USD) and $\Delta_n \widehat{F}_t = \widehat{F}_t - \widehat{F}_{t+n\Delta t}$ (in BTC)

Why is $\Delta_n \widehat{F}_t$ [start − end] price? So that a short inverse perpetual position profits from a decrease in its USD value

Need to multiply $-\theta\,\Delta_n \widehat{F}_t$ by $S_{t+n\Delta t}$ to convert into USD

Realised P&L on the hedged portfolio is $\Delta_N S_t - \theta\,\Delta_N \widehat{F}_t \cdot S_{t+N\Delta t}$

Basic hedging problem is to choose θ to minimize:

$$\sigma^2_{\Delta h}(\theta) := \mathbb{V}\mathrm{ar}\big(\Delta_N S_t - \theta\,\Delta_N \widehat{F}_t\, S_{t+N\Delta t}\big)$$

Margins and Liquidations

Hedger deposits b bitcoin in margin account $(0 < b < 1) \Rightarrow$ leverage is b^{-1}

So $b =$ initial margin requirement (e.g. 1%) plus buffer

Margin account marked each Δt so that at time $t + n\Delta t$:

- $\Delta_n \widehat{F} < 0 \Rightarrow$ marked gain $\theta\Delta_n \widehat{F}_t$ from t to $t + n\Delta t$
- $\Delta_n \widehat{F} > 0 \Rightarrow$ marked loss of $\theta\Delta_n \widehat{F}_t$ from t to $t + n\Delta t$

$\theta\Delta_n \widehat{F}_t$ is cumulative unrealised gain or loss (settlement is daily)

Upper constraint $m = b - M$ on hedgers ability to meet margin calls

M is maintenance margin

Forced liquidation occurs if $\theta\Delta_n \widehat{F}_t > m$ for some n, with $0 < n \leq N$

$\Rightarrow$ Hedger wants to minimize:

$$P(m,\theta) = \mathbb{P}\mathrm{rob}(\text{default event}) := \mathbb{P}\mathrm{rob}\left(\theta \max_{0\leq n\leq N} \Delta_n \widehat{F}_t > m\right)$$

Hedging Problem and Solution

Introduce a default aversion factor γ to obtain final objective:

$$\min_{\theta>0} \quad \{\sigma^2_{\Delta h}(\theta) + \gamma\,\sigma^2_{\Delta S}\,P(m,\theta)\}$$

Solution: $\theta^* = w^{-1}\theta_0$

where $w = S\widehat{F}^2$ at the time of the hedge and θ_0 is positive real root of

$$a(x)x^{-2} + x - b = 0$$

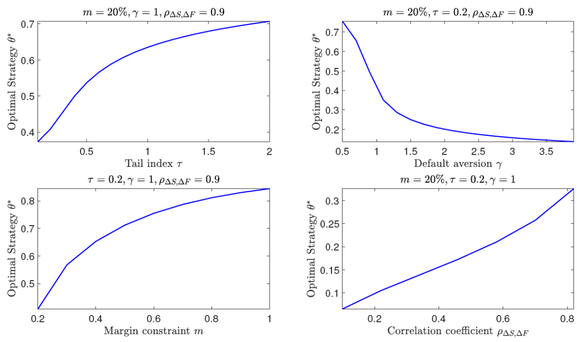
with:

$$a(x) = \frac{\gamma m v^2 w}{2\alpha}\,\frac{\sigma^2_{\Delta S}}{\sigma^2_{\Delta F}}\,\exp\left[-\left(1+\tau\frac{mwx^{-1}-\beta}{\alpha}\right)^{-\frac{1}{\tau}}\right]\left(1+\tau\frac{mwx^{-1}-\beta}{\alpha}\right)^{-\frac{1}{\tau}-1}.$$

Here $v = \sigma_{\Delta S}/\sigma_{\Delta F}$ is the ratio of n-period spot P&L volatility to that of the direct futures $F_t = \widehat{F}_t^{-1}$ and $b = \frac{\sigma_{\Delta S \Delta F}}{\sigma^2_{\Delta F}}$ where the numerator is the covariance.

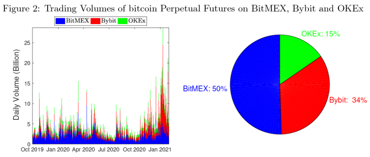
The constants α, β and τ are scale, location and right tail index parameter of the distribution of extreme values of $\Delta\widehat{F}$.

Sensitivity Analysis of Optimal Strategy



Data

Figure 2: Trading Volumes of bitcoin Perpetual Futures on BitMEX, Bybit and OKEx



Note. The left panel plots daily trading volumes (in billion USD) of bitcoin perpetual futures on BitMEX, Bybit and OKEx, while the right panel plots their relative weight of aggregate volumes. For both panels, data spans from October 2019 to January 2021.

Table 3: Data Information of bitcoin Spot and Perpetual Futures

	Exchange	Start Date	End Date	# of Obs.
bitcoin Spot	Bitstamp	2017/1/1	2021/1/31	2,148,480
	Coinbase	2017/1/1	2021/1/31	2,148,480
	Gemini	2017/8/29	2021/1/31	1,822,239
Perpetual Futures	BitMEX	2017/1/1	2021/1/31	2,148,480
	Bybit	2019/10/1	2021/1/31	704,160
	OKEx	2019/7/18	2021/1/31	812,160

Note. This table reports the start/end date and the number of observations for each of the three spot and three futures price data sources used in the empirical analysis. The difference in the start date is mainly due to the availability of the futures contracts or data.

Correlations

Table 4: Correlations of Spot and Perpetuals 1-min Price Changes

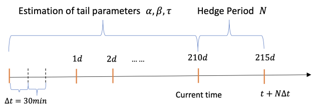
		Spot			Futures		
	Variables	Bitstamp	Coinbase	Gemini	BitMEX	Bybit	OKEx
Spot	Bitstamp	1.00	0.77	0.74	0.79	0.90	0.90
	Coinbase	0.77	1.00	0.79	0.84	0.95	0.96
	Gemini	0.74	0.79	1.00	0.78	0.88	0.87
Perpetuals	BitMEX	0.79	0.84	0.78	1.00	0.95	0.96
	Bybit	0.90	0.95	0.88	0.95	1.00	0.95
	OKEx	0.90	0.96	0.87	0.96	0.95	1.00

Note. This table reports the correlation coefficient between ΔS and ΔF, and between themselves for different choices of bitcoin spot and perpetual futures exchanges. All the results are computed using the 1-min data over the full available sample period (see Table 3).

Estimation of Tail Parameters

Consider results for 30-minute monitoring and hedge horizon of 5 days

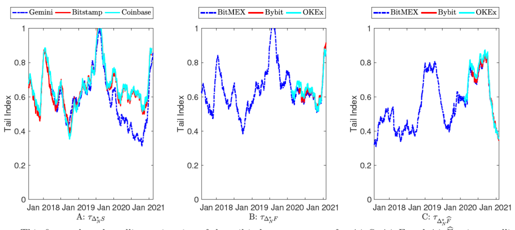
We estimate GED parameters based on 210 daily extreme values assumed to be maximum of $\left(\Delta\widehat{F}_{t+\Delta t}, \Delta\widehat{F}_{t+2\Delta t}, \cdots, \Delta\widehat{F}_{t+48\Delta t}\right)$



Most important parameter for optimal hedge ratio is right tail index

Right Tail Index Results

Figure 3: Rolling Estimation of τ for $\Delta_N^* S$, $\Delta_N^* F$, and $\Delta_N^* \widehat{F}$



Note. This figure plots the rolling estimation of the tail index parameter τ for $\Delta_N^* S$, $\Delta_N^* F$ and $\Delta_N^* \widehat{F}$ using a rolling window of 210 days. The full sample data are specified in Table 3. For illustration we set the monitoring frequency $\Delta t = 1$ minute and the hedge horizon $N\Delta t = 1$ day. Panel A plots the estimated τ of $\Delta_N^* S$ for Bitstamp, Coinbase and Gemini. Panels B to C plot the estimated τ's of $\Delta_N^* F$ and $\Delta_N^* \widehat{F}$ for the bitcoin perpetual futures traded on BitMEX, Bybit and OKEx, respectively.

Hedging Effectiveness

Table 6: Hedge Effectiveness (%) of bitcoin Perpetual Futures

		Futures	BitMEX			Bybit			OKEx		
Spot	m		8h	1d	2d	8h	1d	2d	8h	1d	2d
Bistamp	0.1	$\gamma = 20$	69.67	43.28	30.37	78.57	49.06	38.09	82.75	55.44	40.71
		$\gamma = 50$	49.76	22.13	19.17	58.16	26.83	19.15	63.14	29.53	21.58
		$\gamma = 200$	25.15	18.75	10.25	28.19	11.23	11.06	32.95	15.52	13.01
	0.2	$\gamma = 20$	88.86	73.57	59.33	95.04	81.93	68.06	97.73	86.39	73.91
		$\gamma = 50$	77.89	55.22	42.27	88.18	61.21	50.23	91.83	66.22	54.16
		$\gamma = 200$	52.10	32.32	18.17	63.17	29.32	25.76	68.39	29.99	26.72
	0.5	$\gamma = 20$	97.51	93.77	86.93	97.65	96.28	95.60	99.79	99.12	97.97
		$\gamma = 50$	94.20	85.78	75.01	97.35	93.00	89.29	99.59	96.23	92.74
		$\gamma = 200$	83.04	64.59	52.98	94.18	73.79	67.79	97.10	77.65	72.40
Coinbase	0.1	$\gamma = 20$	70.32	44.19	30.37	78.62	49.21	38.79	82.73	55.47	32.34
		$\gamma = 50$	50.34	25.37	18.95	58.19	26.90	19.78	63.11	29.55	20.47
		$\gamma = 200$	22.46	18.53	8.45	28.20	21.24	12.36	33.79	21.38	13.27
	0.2	$\gamma = 20$	88.98	70.07	59.42	95.11	82.01	68.37	97.74	86.46	73.79
		$\gamma = 50$	78.17	50.23	41.32	88.24	61.32	50.40	91.83	66.33	54.08
		$\gamma = 200$	52.47	32.93	24.41	63.21	29.41	26.01	68.37	30.02	26.56
	0.5	$\gamma = 20$	97.49	93.20	86.26	97.73	96.30	95.67	99.80	99.12	97.94
		$\gamma = 50$	94.34	84.84	74.65	97.42	93.04	89.46	99.60	96.26	92.68
		$\gamma = 200$	83.50	62.96	54.77	94.26	73.89	68.01	97.11	77.76	72.29
Gemini	0.1	$\gamma = 20$	82.75	58.82	42.97	78.37	48.96	38.03	82.48	55.53	42.54
		$\gamma = 50$	62.66	42.58	29.59	57.93	26.97	18.68	62.84	30.14	21.13
		$\gamma = 200$	31.57	23.22	12.36	28.60	21.27	11.25	33.56	21.50	12.44
	0.2	$\gamma = 20$	96.99	88.99	75.19	94.97	81.92	68.28	97.61	86.35	73.81
		$\gamma = 50$	90.66	75.91	58.68	88.04	61.35	50.34	91.64	66.34	54.07
		$\gamma = 200$	65.42	51.49	36.46	62.95	29.44	26.04	68.13	30.13	26.82
	0.5	$\gamma = 20$	99.13	99.56	98.02	97.62	96.19	95.57	99.70	99.00	97.88
		$\gamma = 50$	98.79	98.20	93.39	97.31	92.94	89.35	99.49	96.14	92.64
		$\gamma = 200$	95.05	90.17	77.20	94.11	73.91	67.89	96.97	77.76	72.39

Note. This table reports the hedge effectiveness, defined in (10), of the optimal strategy θ^* at different hedge horizon $N\Delta t$, margin constraint m and default aversion γ. We consider three bitcoin spot markets Bitstamp, Coinbase and Gemini, and three bitcoin perpetual futures on BitMEX, Bybit and OKEx. We set monitoring frequency $\Delta t = 1$min.

6. Speculation with Crypto Perpetuals

Average of 12-hour volume and open interest (both in \$ billion) and forced liquidations (\$ million)

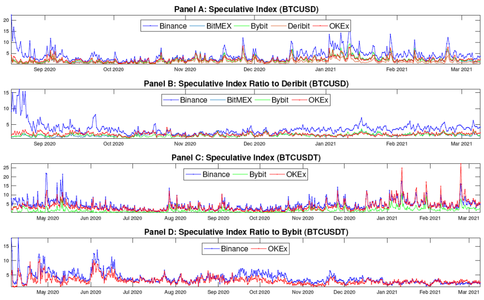
		USD Denominated Bitcoin Perpetuals				
		Binance	BitMEX	Bybit	Deribit	OKEx
Mean	Volume(B)	1.06	1.09	1.20	0.24	0.37
	OI(B)	0.22	0.61	0.55	0.18	0.16
	Short Liq.(M)	3.86	5.12	3.46	0.54	1.34
	Long Liq.(M)	1.44	9.80	6.96	0.67	0.97
	Liquidation(M)	5.30	14.92	10.42	1.21	2.31
Median	Volume(B)	0.37	0.92	0.58	0.12	0.27
	OI(B)	0.15	0.59	0.37	0.11	0.14
	Short Liq.(M)	0.86	1.74	1.08	0.10	0.40
	Long Liq.(M)	0.36	2.38	1.54	0.14	0.39
	Liquidation(M)	1.22	4.12	2.62	0.23	0.80
	Start Period	2020/8/17	2020/1/29	2020/4/4	2020/3/7	2020/3/28
	End Period	2021/3/9	2021/3/9	2021/3/9	2021/3/9	2021/3/9

USD Perpetuals

Average of 12-hour volume and open interest (both in \$ billion) and forced liquidations (\$ million)

		USDT Denominated Bitcoin Perpetuals		
		Binance	Bybit	OKEx
Mean	Volume(B)	2.53	0.20	0.36
	OI(B)	0.54	0.11	0.09
	Short Liq.(M)	10.89	1.82	2.00
	Long Liq.(M)	14.63	2.19	1.60
	Liquidation(M)	25.52	4.01	3.60
Median	Volume(B)	1.40	0.07	0.23
	OI(B)	0.40	0.07	0.09
	Short Liq.(M)	3.55	0.47	0.60
	Long Liq.(M)	4.33	0.47	0.53
	Liquidation(M)	7.88	0.94	1.14
	Start Period	2020/1/26	2020/4/4	2020/3/28
	End Period	2021/3/9	2021/3/9	2021/3/9

VOL/OI USD Perpetuals Every 12 hrs



Note. Panel A plots USD denominated bitcoin perpetual speculative index for Binance, BitMEX, Bybit, Deribit, and OKEx. Panel B plots the speculative index ratios to Deribit. The USD denominated $\mathcal{SI}$ indices of BitMEX, Bybit and OKEx are about double of Deribit and Binance is nearly four times of Deribit.

Table 2: Summary Statistics for Bitcoin Perpetuals Speculative Index

USD Denominated Bitcoin Perpetuals	Full Sample					Common Date: 2020/8/17-2021/3/9				
	Binance	BitMEX	Bybit	Deribit	OKEx	Binance	BitMEX	Bybit	Deribit	OKEx
min	0.53	0.26	0.21	0.13	0.41	0.53	0.39	0.32	0.19	0.43
p25	2.23	0.99	1.12	0.63	1.33	2.23	1.14	1.12	0.66	1.40
median	3.43	1.55	1.74	1.01	1.89	3.43	1.74	1.69	1.02	1.91
mean	4.11	1.89	2.08	1.29	2.19	4.11	2.06	2.03	1.23	2.23
p75	5.02	2.39	2.48	1.54	2.62	5.02	2.52	2.38	1.49	2.62
max	22.31	10.21	11.99	8.69	10.75	22.31	7.72	9.46	5.03	8.30
std	2.84	1.34	1.53	1.05	1.32	2.84	1.32	1.45	0.83	1.29
Obs	408	811	678	734	693	408	408	408	408	408

USDT Denominated Bitcoin Perpetuals	Full Sample					Common Date: 2020/4/4-2021/3/9				
	Binance	BitMEX	Bybit	Deribit	OKEx	Binance	BitMEX	Bybit	Deribit	OKEx
min	0.60		0.15		0.52	0.60		0.15		0.52
p25	2.52		0.56		1.84	2.44		0.56		1.79
median	3.85		1.04		2.86	3.68		1.04		2.82
mean	4.72		1.46		3.63	4.36		1.46		3.62
p75	5.69		1.91		4.48	5.38		1.91		4.48
max	30.27		10.90		27.26	21.93		10.90		27.26
std	3.52		1.35		2.84	2.87		1.35		2.87
Obs	818		678		693	678		678		678

Note. This table reports summary statistics for USD and USDT denominated bitcoin perpetual speculative index on Binance, BitMEX, Bybit, Deribit, and OKEx exchanges. The USD speculative indices of BitMEX, Bybit and OKEx are almost double of Deribit and Binance is nearly four times of Deribit.

Binance USDT Perpetuals

Common Date: 2020/8/22 - 2021/3/9

	ADA	BCH	BNB	BTC	DOGE	DOT	ETH	LTC	TRX	XRP
min	0.57	0.56	0.47	0.98	0.25	0.31	0.99	0.35	0.34	0.39
p25	1.53	1.57	1.11	2.60	0.93	1.18	2.32	1.76	1.05	1.36
median	2.27	2.35	1.61	3.86	1.85	1.91	3.24	2.65	1.53	2.23
mean	2.91	2.97	2.34	4.39	4.99	2.38	3.76	3.17	2.29	3.71
p75	3.50	3.69	2.63	5.21	4.23	2.88	4.55	4.05	2.52	4.19
max	6.65	13.91	16.53	6.85	58.50	11.66	10.40	6.59	37.69	14.31
std	2.07	2.20	2.26	2.61	11.30	1.79	2.20	2.02	2.70	4.29
Obs	398	398	398	398	398	398	398	398	398	398

Binance USD Perpetuals

Common Date: 2020/9/9 - 2021/3/9

	ADA	BCH	BNB	BTC	DOT	ETH	LTC	TRX	XRP
min	0.77	0.38	0.56	0.53	0.21	0.74	0.07	0.17	0.09
p25	2.92	1.18	1.79	2.15	0.99	1.91	1.44	1.08	1.49
median	4.46	1.79	2.64	3.32	1.58	2.71	2.37	1.87	3.37
mean	5.62	3.83	3.35	3.94	2.01	3.26	4.44	2.77	6.67
p75	6.91	3.26	4.29	4.91	2.56	3.91	3.73	3.09	6.99
max	33.85	87.18	15.60	16.27	10.05	17.36	113.32	23.09	138.79
std	4.28	7.58	2.31	2.61	1.54	2.17	11.14	3.05	12.28
Obs	332	332	332	332	332	332	332	332	332

Correlation of VOL/OI Between USDT Perpetuals

	ADA	BCH	BNB	BTC	DOGE	DOT	ETH	LTC	TRX	XRP
ADA	1.00	0.51	0.40	0.54	0.21	0.25	0.62	0.51	0.26	0.42
BCH	0.51	1.00	0.25	0.65	0.20	0.24	0.70	0.73	0.29	0.36
BNB	0.40	0.25	1.00	0.22	0.09	0.17	0.20	0.16	0.17	0.09
BTC	0.54	0.65	0.22	1.00	0.33	0.26	0.81	0.79	0.27	0.32
DOGE	0.21	0.20	0.09	0.33	1.00	0.05	0.24	0.19	0.10	0.20
DOT	0.25	0.24	0.17	0.26	0.05	1.00	0.45	0.18	0.41	0.00
ETH	0.62	0.70	0.20	0.81	0.24	0.45	1.00	0.73	0.44	0.31
LTC	0.51	0.73	0.16	0.79	0.19	0.18	0.73	1.00	0.24	0.51
TRX	0.26	0.29	0.17	0.27	0.10	0.41	0.44	0.24	1.00	0.21
XRP	0.42	0.36	0.09	0.32	0.20	0.00	0.31	0.51	0.21	1.00

Correlation of VOL/OI Between USD Perpetuals

	ADA	BCH	BNB	BTC	DOT	ETH	LTC	TRX	XRP
ADA	1.00	0.18	0.28	0.45	0.35	0.53	0.21	0.38	0.24
BCH	0.18	1.00	0.39	-0.02	0.08	0.06	0.91	0.44	0.56
BNB	0.28	0.39	1.00	0.19	0.17	0.19	0.32	0.41	0.20
BTC	0.45	-0.02	0.19	1.00	0.44	0.82	0.08	0.17	0.15
DOT	0.35	0.08	0.17	0.44	1.00	0.54	0.09	0.16	0.08
ETH	0.53	0.06	0.19	0.82	0.54	1.00	0.13	0.25	0.16
LTC	0.21	0.91	0.32	0.08	0.09	0.13	1.00	0.33	0.71
TRX	0.38	0.44	0.41	0.17	0.16	0.25	0.33	1.00	0.28
XRP	0.24	0.56	0.20	0.15	0.08	0.16	0.71	0.28	1.00