

**TATINE**

Solutions for Credit Risk  
Measurement

# Multi-Factor Model Applied to SMEs

Barcelona, February 2013

- Research · Consulting · Advanced Tools -

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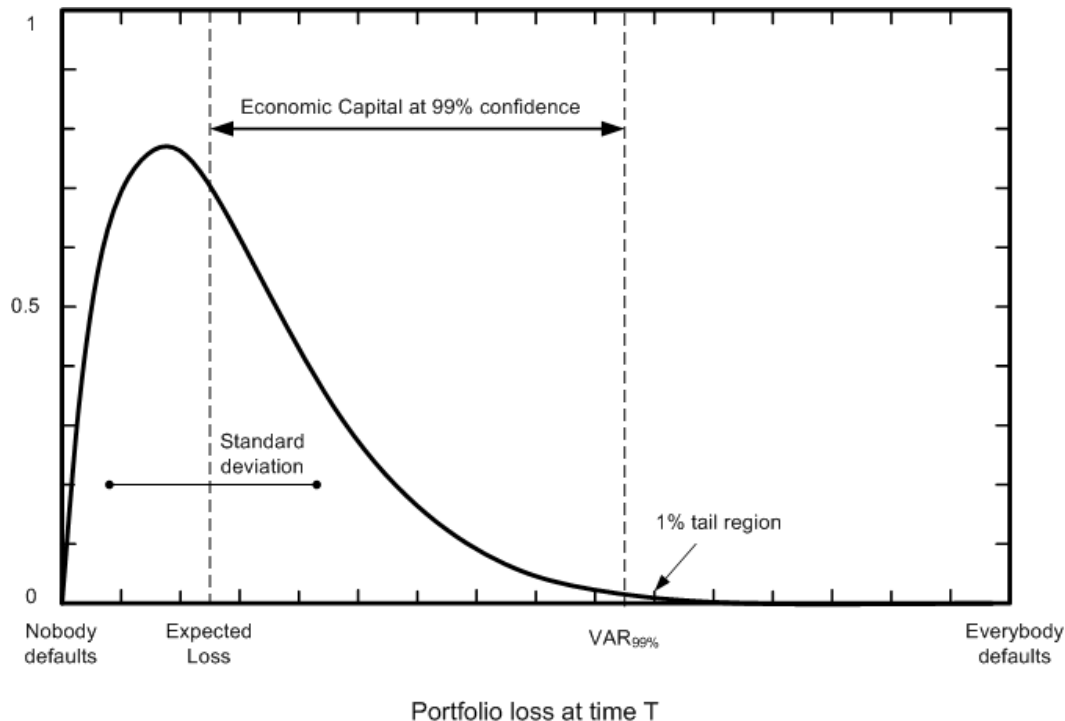
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- Introduction
- Model Calibration
- Risk Assessment
- Some Results

# Introduction - Objective

Credit risk assessment (eg. VaR or ES)  
of the portfolio of SMEs



- Avoiding oversimplifications
- Considering correlation accurately
- Considering the parameter uncertainty in risk assessment

# Introduction - Portfolio of SMEs



## Features:

- Not-so-large num obligors
- Non Homogeneous
- Economic sectors
- Large amounts
- Variety of products
- There is a PD model
- There is a LGD model
- Unlisted companies
- Historical records of observed defaults of type count-data by sector

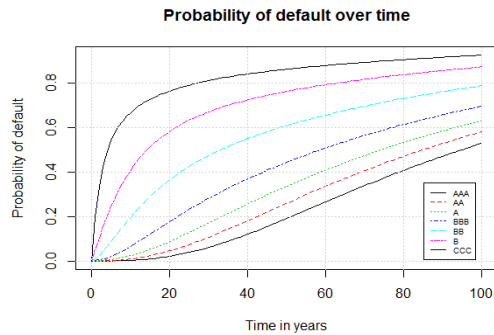
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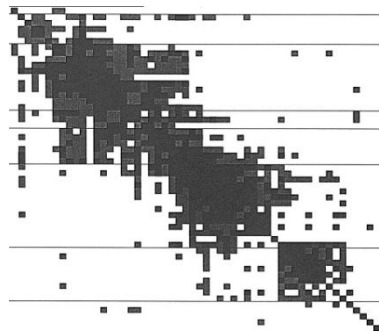
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# Model Calibration - Assumptions



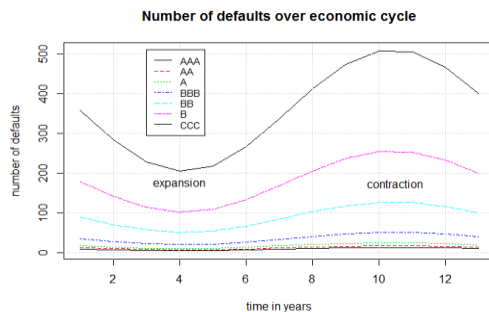
1.

Every company has assigned a probability of default (PD)



2.

The correlation depends only of economic sector



3.

The correlation remains constant over time

# Model Calibration

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Model  $\rightarrow$  Multivariate Obligor Default Times

$$X_i^j = \sqrt{\frac{\nu}{S}} \cdot \left( w_i \cdot Z_i + \sqrt{1 - w_i^2} \cdot \varepsilon_i^j \right) \quad \text{where} \quad \left\{ \begin{array}{l} i = 1, \dots, k \\ j = 1, \dots, n_i \\ w_i \in (0, 1) \quad \forall i \\ 2 \leq \nu \\ Z \sim N(0, R) \\ S \sim \chi_\nu^2 \\ \varepsilon_i^j \sim N(0, 1) \text{ i.i.d } \forall i, j \\ Z, S, \varepsilon_i^j \text{ independents } \forall i, j \end{array} \right.$$

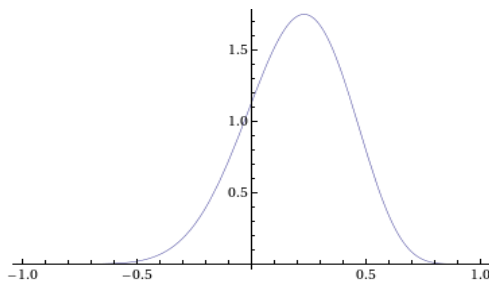
It is the classic multi-factor Merton model extended to the t-Student case.

# Model Calibration

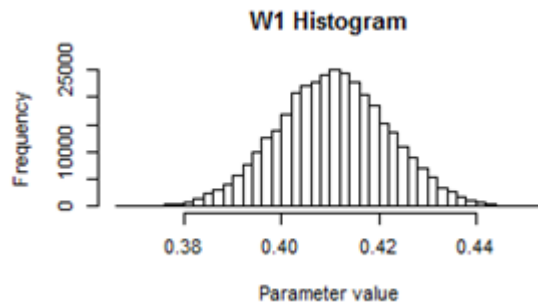
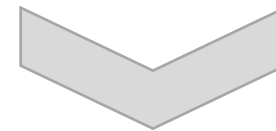
Each observation has the form:  $K = (K_{1j}, \dots, K_{kr})$   
where  $K_{ij} = \#$  defaults in sector  $i$  with rating  $j$  [ in 1 year ]

Parameters to estimate:  $w_i, R, v$

Distr. correl 20  $N(0,0.2)$



- Correlations are difficult to estimate
- Latent variables involved
- Reduced number of observations
- Observations of type Count-Data



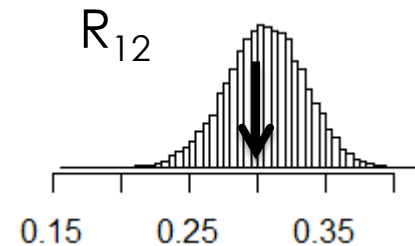
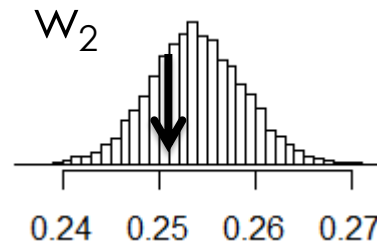
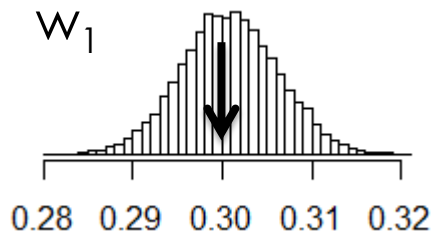
Bayesian estimation + MCMC



# Model Calibration – Example (I)

1. We fix the parameters:  $v, w_i, R$
2. We simulate 1000 observations
3. We estimate the parameters  $w_i', R'$
4.  $w_i, R$  and  $w_i', R'$  are similar

Yes, it works!

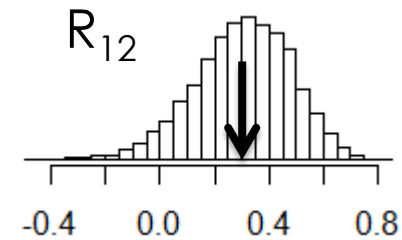
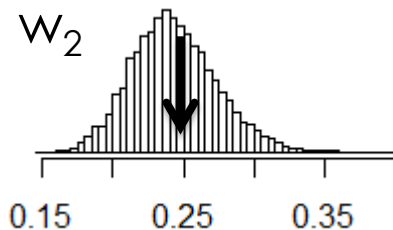
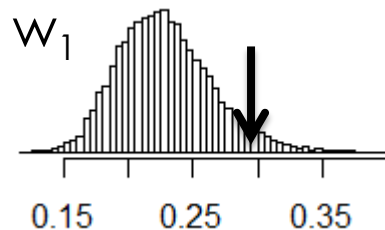


- 2 factors
- 4000 obligors x rating
- $w_1=0.3, w_2=0.25, R_{12}=0.3$
- 100.000 MH sims
- 4 ratings
- 6000 obligors x factor
- $R_{12}$  generated = 0.3027

# Model Calibration – Example (II)

1. We fix the parameters:  $v, w_i, R$
2. We simulate 20 observations
3. We estimate the parameters  $w_i', R'$
4.  $w_i, R$  and  $w_i', R'$  are similar ?

Works,  
but ...



- 2 factors
- 4000 obligors x rating
- $w_1=0.3, w_2=0.25, R_{12}=0.3$
- 150.000 MH sims
- 4 ratings
- 6000 obligors x factor
- $R_{12}$  generated = 0.34

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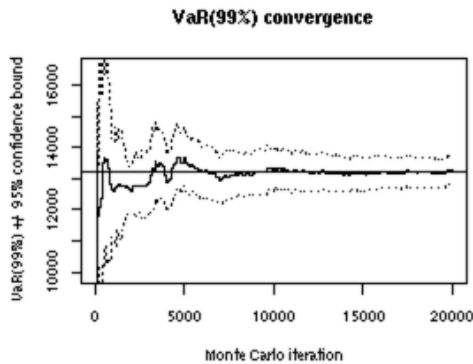
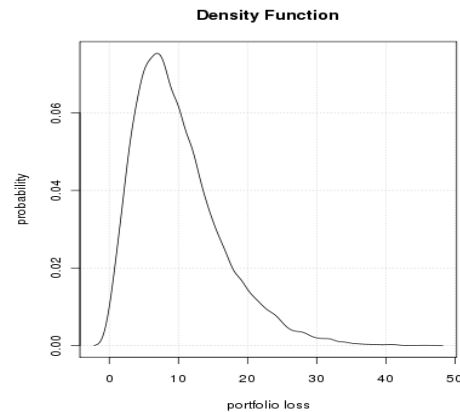
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# Risk Assessment

$$\text{PortfolioLoss} = \sum \text{ObligorLoss}$$

$$\text{ObligorLoss} = \sum \text{AssetLoss}$$

$$\text{AssetLoss} = \begin{cases} \text{exposure}(t) \cdot [1 - \text{recovery}(t)] & \text{if } t \leq T \\ 0 & \text{if } t > T \end{cases}$$



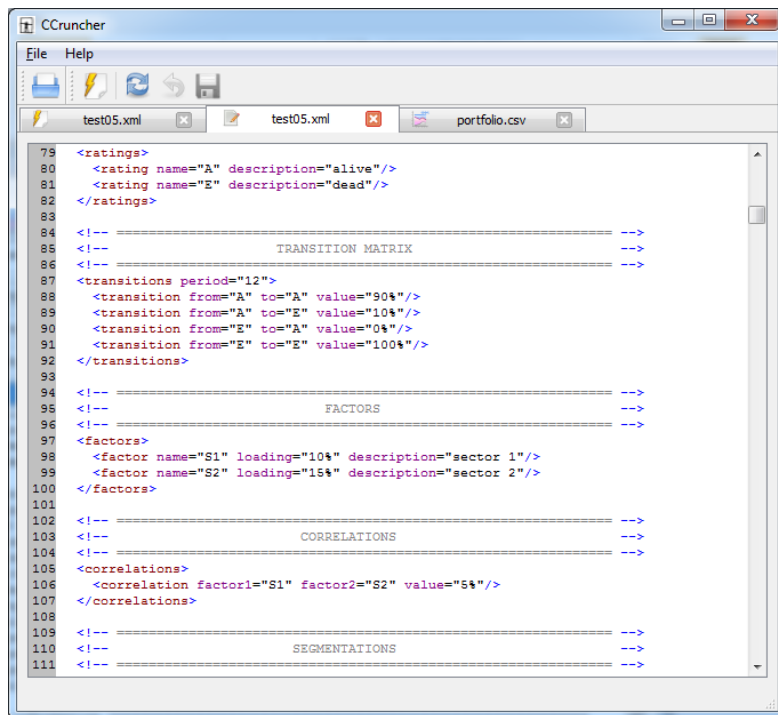
## Monte Carlo method

- Default times simulation
- Portfolio Loss distribution
- Risk valuation (VaR, ES)

Risk aggregation can be analyzed

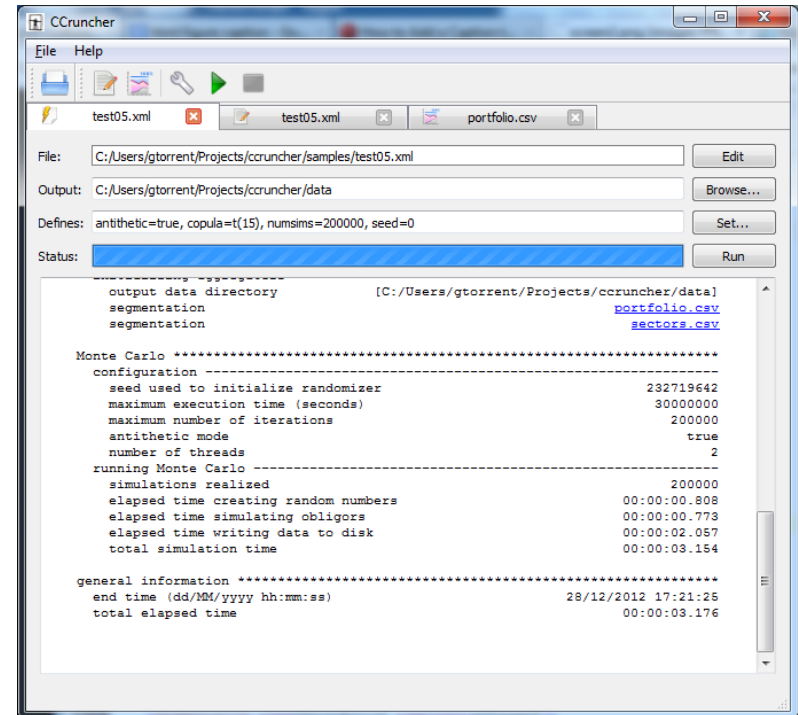
# Risk Assessment

## Problem Formulation (XML)



```
79 <ratings>
80 <rating name="A" description="alive"/>
81 <rating name="E" description="dead"/>
82 </ratings>
83
84 <!-- ===== -->
85 <!-- TRANSITION MATRIX -->
86 <!-- ===== -->
87 <transitions period="12">
88 <transition from="A" to="A" value="90"/>
89 <transition from="A" to="E" value="10"/>
90 <transition from="E" to="A" value="04"/>
91 <transition from="E" to="E" value="100"/>
92 </transitions>
93
94 <!-- ===== -->
95 <!-- FACTORS -->
96 <!-- ===== -->
97 <factors>
98 <factor name="S1" loading="10" description="sector 1"/>
99 <factor name="S2" loading="15" description="sector 2"/>
100 </factors>
101
102 <!-- ===== -->
103 <!-- CORRELATIONS -->
104 <!-- ===== -->
105 <correlations>
106 <correlation factor1="S1" factor2="S2" value="5"/>
107 </correlations>
108
109 <!-- ===== -->
110 <!-- SEGMENTATIONS -->
111 <!-- ===== -->
```

## Problem Simulation (MC)



```
File: C:/Users/gtorrent/Projects/ccruncher/samples/test05.xml Edit
Output: C:/Users/gtorrent/Projects/ccruncher/data Browse...
Defines: antithetic=true, copula=t(15), numsim=200000, seed=0 Set...
Status: Run

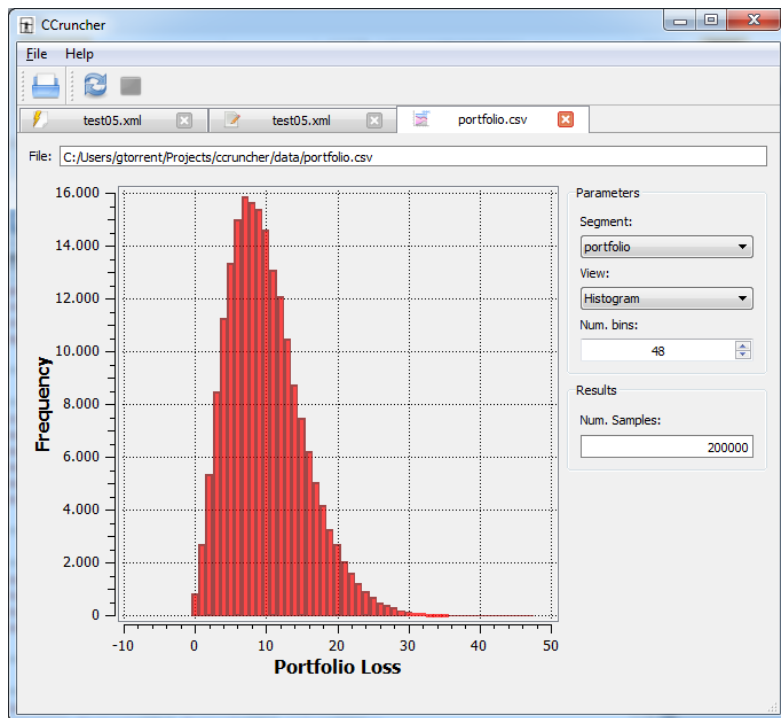
output data directory [C:/Users/gtorrent/Projects/ccruncher/data]
segmentation portfolio.csv
segmentation sectors.csv

Monte Carlo -----
configuration -----
seed used to initialize randomizer 232719642
maximum execution time (seconds) 3000000
maximum number of iterations 200000
antithetic mode true
number of threads 2
running Monte Carlo -----
simulations realized 200000
elapsed time creating random numbers 00:00:00.808
elapsed time simulating obligors 00:00:00.773
elapsed time writing data to disk 00:00:02.057
total simulation time 00:00:03.154

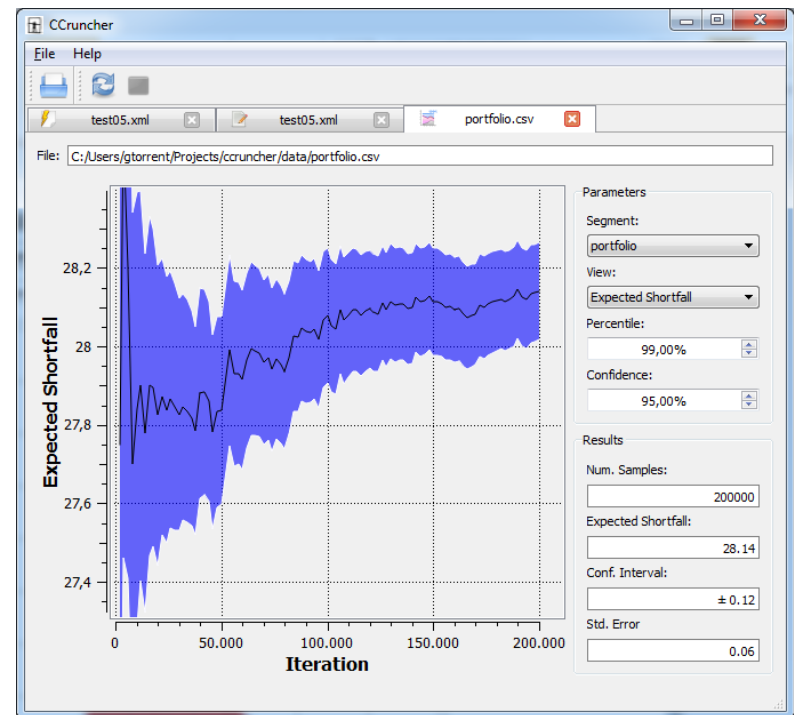
general information -----
end time (dd/MM/yyyy hh:mm:ss) 28/12/2012 17:21:25
total elapsed time 00:00:03.176
```

# Risk Assessment

## Data Analysis (I)



## Data Analysis (II)



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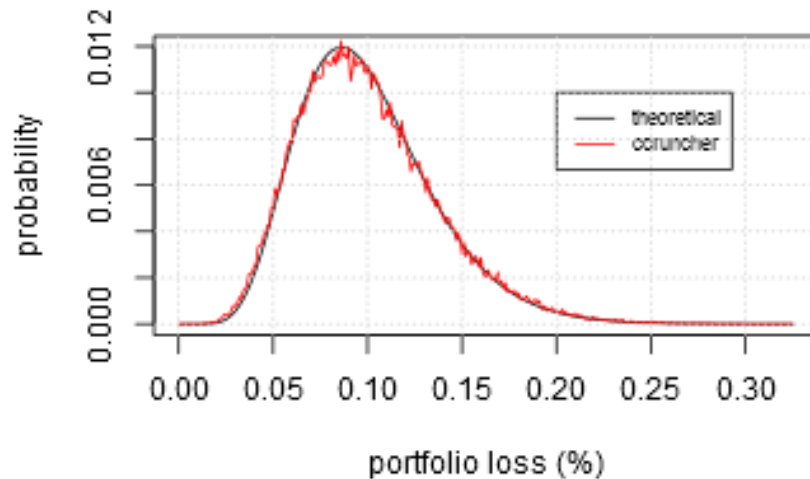


# Some Results – LHP Replication

A simple case: The Large Homogenous Pool

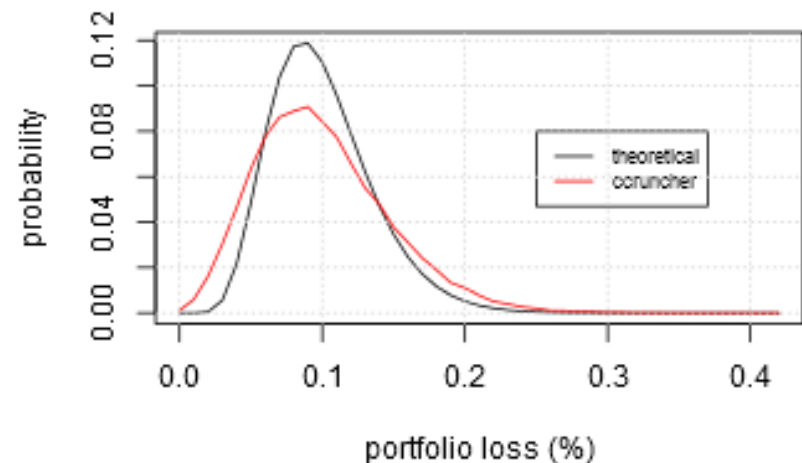
1000 obligors ( $\sim\infty$ )

**Portfolio Loss Distribution**



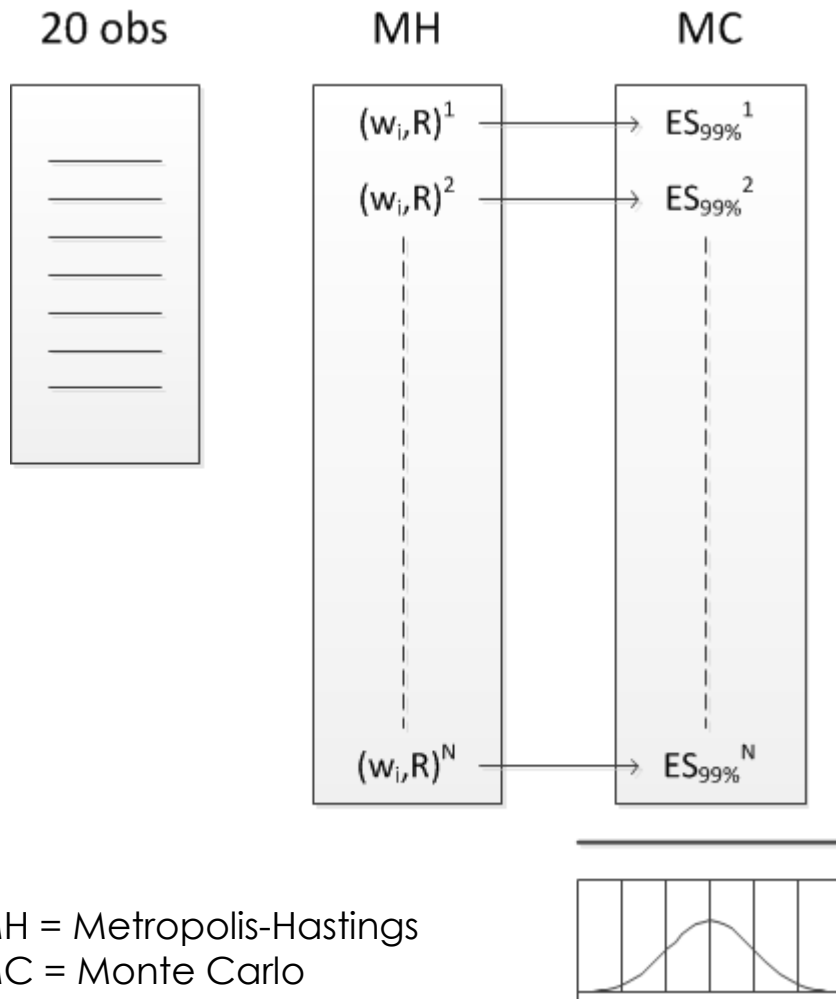
100 obligors

**100 obligors**





# Some Results – Params Uncertainty (I)



MH = Metropolis-Hastings  
MC = Monte Carlo

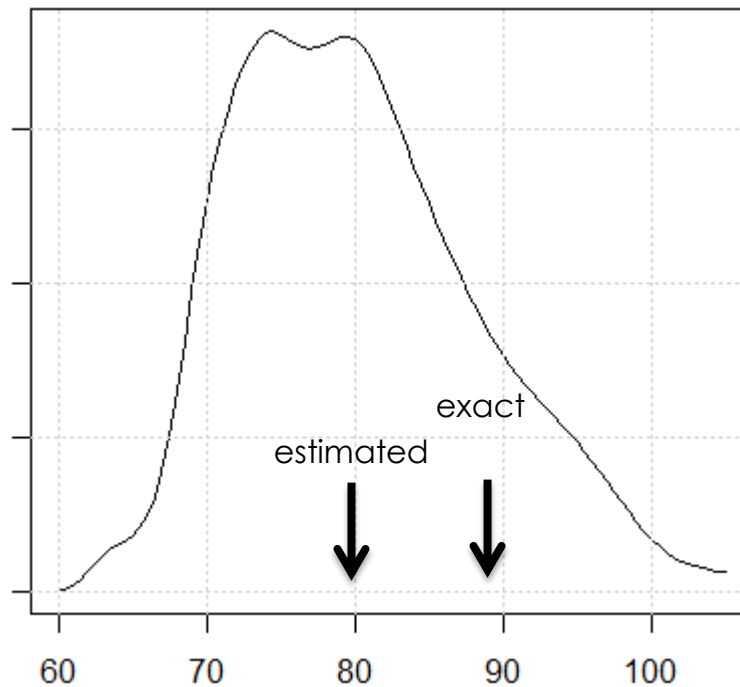
Credit Risk estimator (eg.  $ES_{99\%}$ ) can be considered a distribution instead of a fixed value.

The stochastic behavior is due to parameter uncertainty.

# Some Results – Params Uncertainty (II)

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Expected Shortfall 99%



N = 155

exact = parameters used to simulate observations

estimated = parameters estimated from 20 observations

# Some Results – Others

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Download the software from <http://ccruncher.net>  
Use it to analyze your portfolio !

- Define your risk statistics
- How many factors ?
- Exposures/Recoveries impact
- Risk disaggregation
- Stress-Testing
- ...

# Conclusions

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## CURRENT

- Exact parameters
- Simplified model
- Basic computation

## PROPOSAL

- Fuzzy parameters
- Accurate model
- Intensive computation

[www.tatine.es](http://www.tatine.es)

[info@tatine.es](mailto:info@tatine.es)