TATINE | Solutions for Credit Risk Measurement

Multi-Factor Model
Applied to SMEs

Barcelona, February 2013

- Research · Consulting · Advanced Tools -

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- Introduction
- Model Calibration
- Risk Assessment
- Some Results

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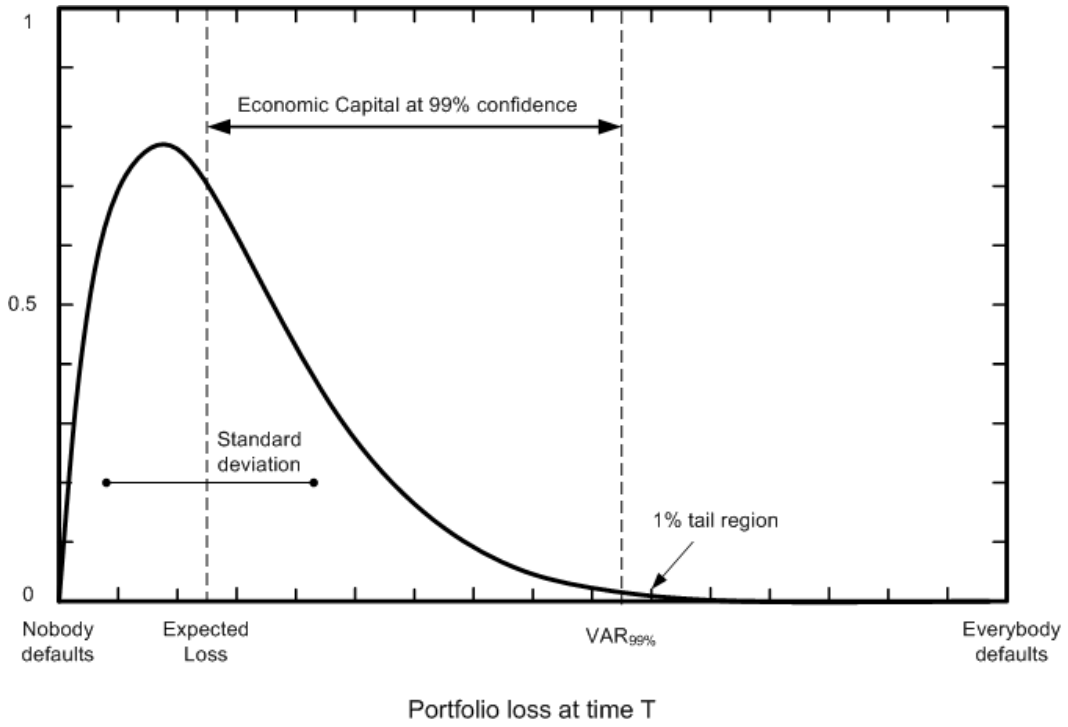
Introduction - Objective

Credit risk assessment (eg. VaR or ES)
of the portfolio of SMEs



- Avoiding oversimplifications
- Considering correlation accurately
- Considering the parameter uncertainty in risk assessment

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Introduction - Portfolio of SMEs

Features:

- Not-so-large num obligors
- Non Homogeneous
- Economic sectors
- Large amounts
- Variety of products
- There is a PD model
- There is a LGD model
- Unlisted companies
- Historical records of observed defaults of type count-data by sector

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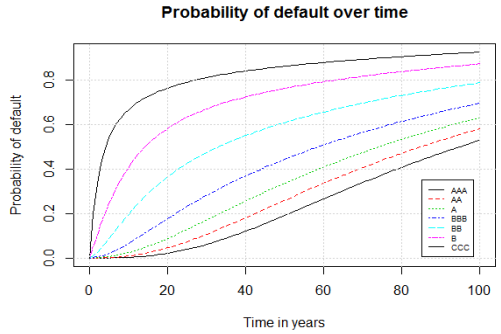
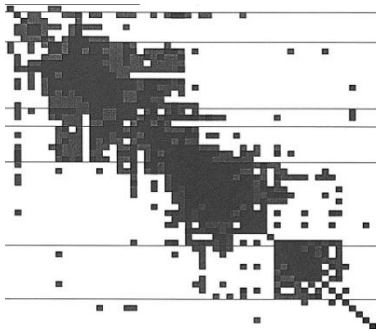
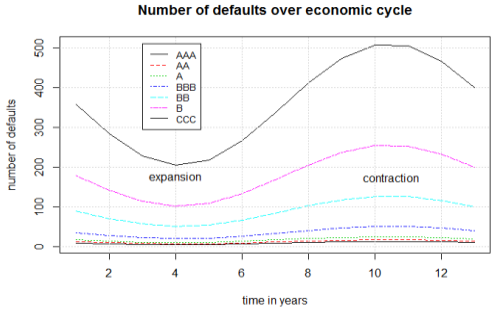
Model Calibration - Assumptions



1. Every company has assigned a probability of default (PD)



2. The correlation depends only of economic sector



3. The correlation remains constant over time

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Model Calibration

Model → Multivariate Obligors Default Times

$$X_i^j = \sqrt{\frac{\nu}{S}} \cdot \left(w_i \cdot Z_i + \sqrt{1 - w_i^2} \cdot \varepsilon_i^j \right) \quad \text{where} \begin{cases} i = 1, \cdots, k \\ j = 1, \cdots, n_i \\ w_i \in (0,1) \ \forall i \\ 2 \leq \nu \\ Z \sim N(0, R) \\ S \sim \chi_\nu^2 \\ \varepsilon_i^j \sim N(0,1) \text{ i.i.d } \forall i, j \\ Z, S, \varepsilon_i^j \text{ independents } \ \forall i, j \end{cases}$$

It is the classic multi-factor Merton model extended to the t-Student case.

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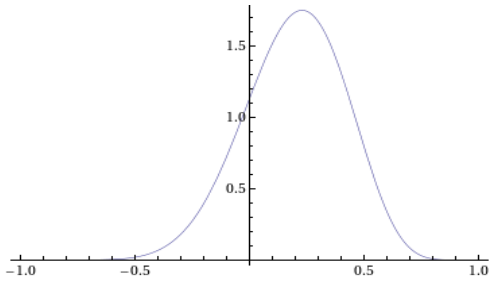
Model Calibration

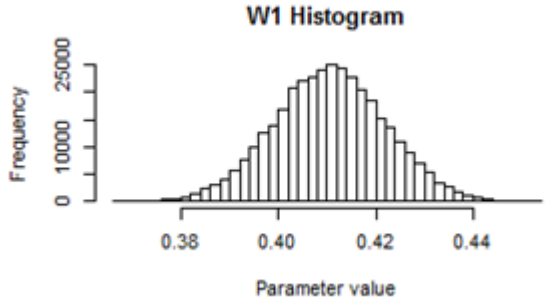
Each observation has the form: $K = (K_{11}, \ldots, K_{kr})$
where K_{ij} = # defaults in sector i with rating j [in 1 year]

Parameters to estimate: w_i, R, ν

Distr. correl 20 N(0,0.2)



- Correlations are difficult to estimate
- Latent variables involved
- Reduced number of observations
- Observations of type Count-Data



Bayesian estimation + MCMC

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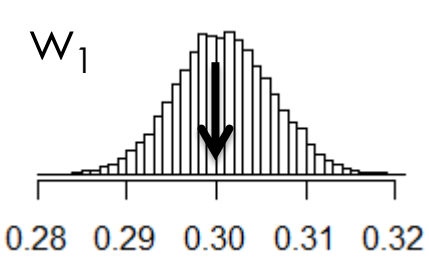
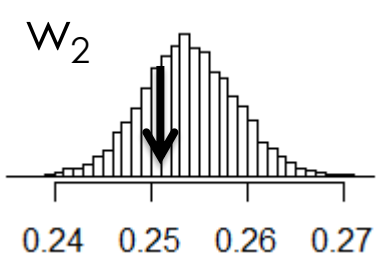
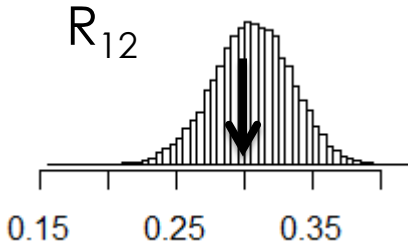
Model Calibration – Example (I)

1. We fix the parameters: v, w_i, R
2. We simulate *1000* observations
3. We estimate the parameters w_i', R'
4. w_i, R and w_i', R' are similar

Yes, it works!







- 2 factors
- 4 ratings
- 4000 obligors x rating
- 6000 obligors x factor
- w_1=0.3, w_2=0.25, R_{12}=0.3
- R_{12} *generated* = 0.3027
- *100.000 MH sims*

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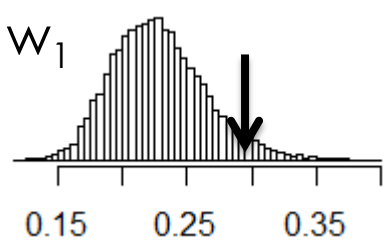
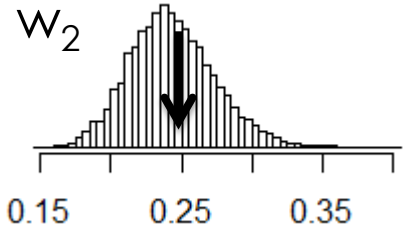
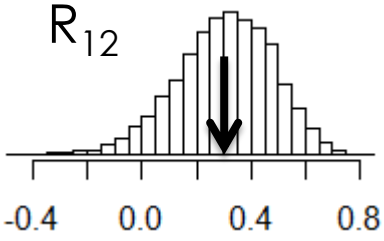
Model Calibration – Example (II)

1. We fix the parameters: v, w_i, R
2. We simulate <u>20</u> observations
3. We estimate the parameters w_i', R'
4. w_i, R and w_i', R' are similar ?

Works, but …







- 2 factors
- 4 ratings
- 4000 obligors x rating
- 6000 obligors x factor
- w_1=0.3, w_2=0.25, R_{12}=0.3
- R_{12} *generated* = 0.34
- *150.000 MH sims*

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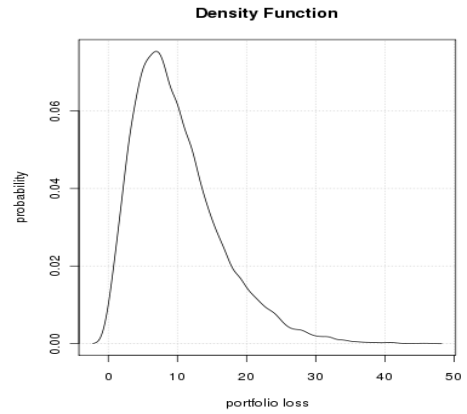
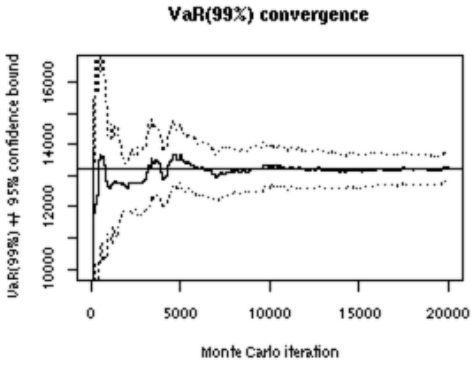
Risk Assessment

$$PortfolioLoss = \sum ObligorLoss$$

$$ObligorLoss = \sum AssetLoss$$

$$AssetLoss = \begin{cases} exposure(t) \cdot [1 - recovery(t)] & if\ t \le T \\ 0 & if\ t > T \end{cases}$$





Monte Carlo method

→ Default times simulation
→ Portfolio Loss distribution
→ Risk valuation (VaR, ES)

Risk aggregation can be analyzed

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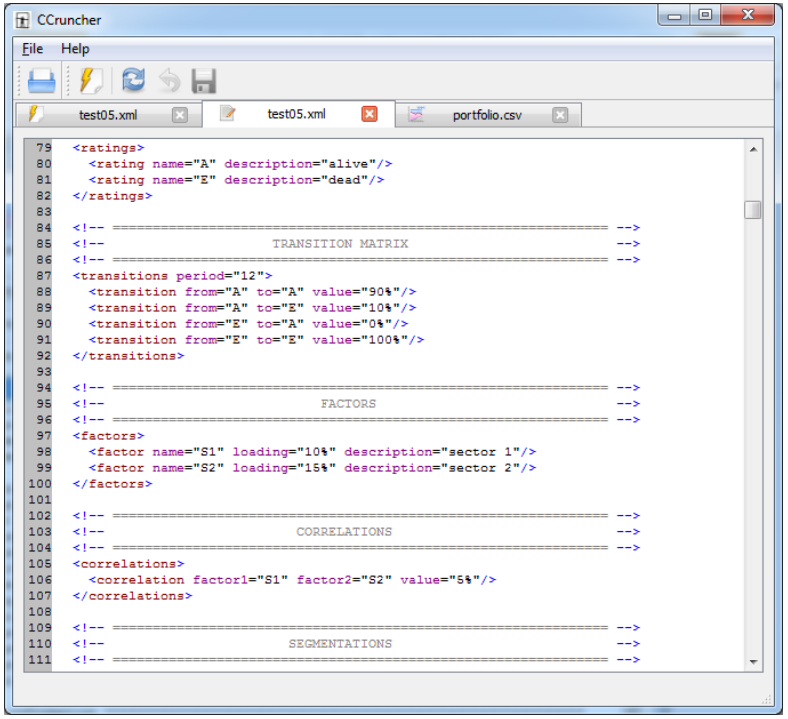
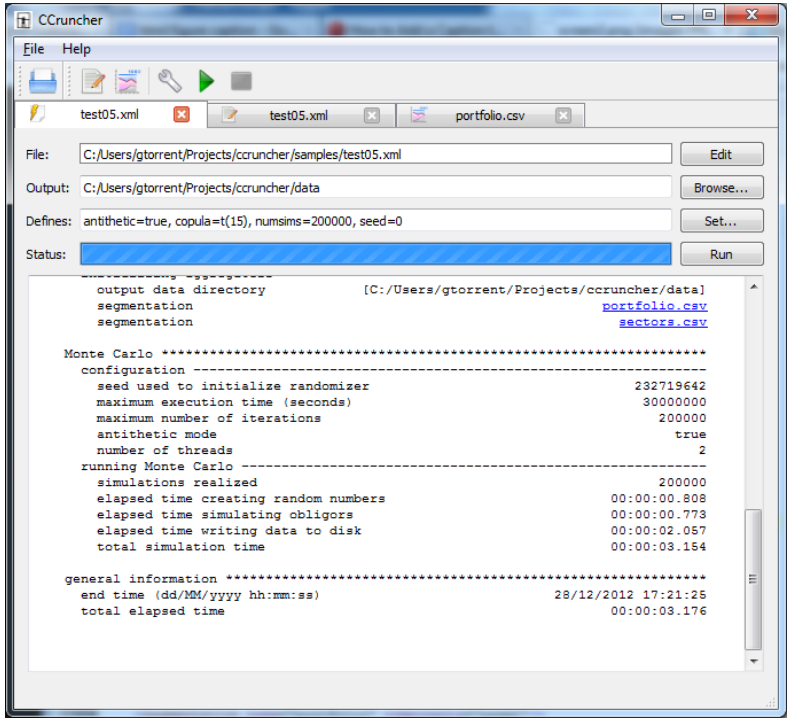
Risk Assessment

Problem Formulation (XML)



Problem Simulation (MC)



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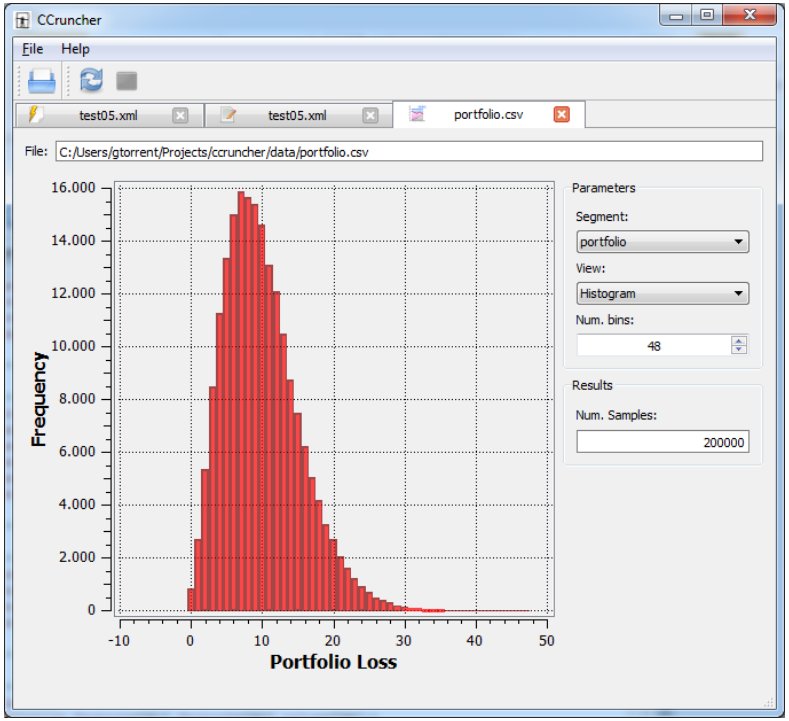
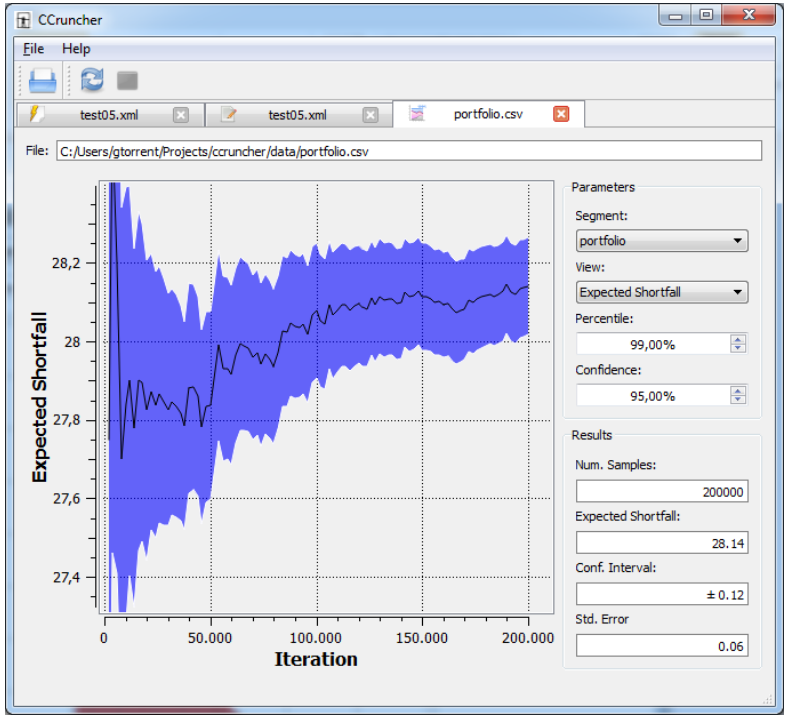
Risk Assessment

Data Analysis (I)



Data Analysis (II)



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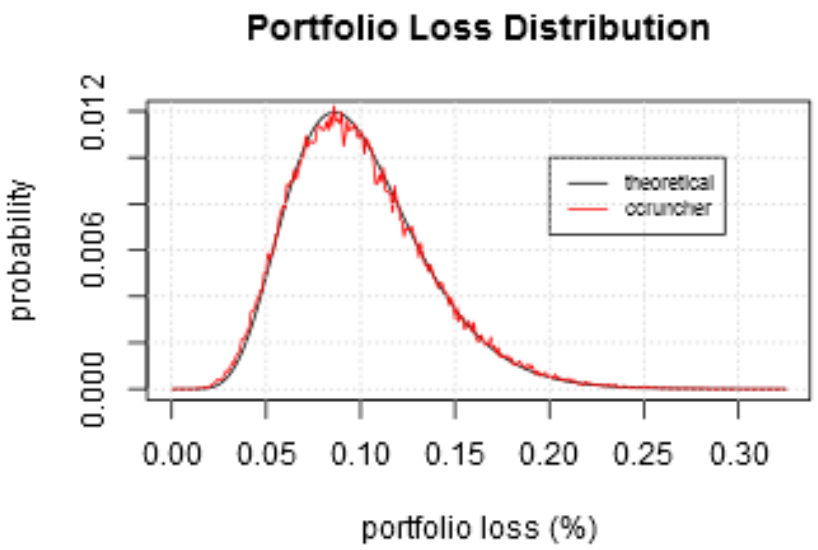
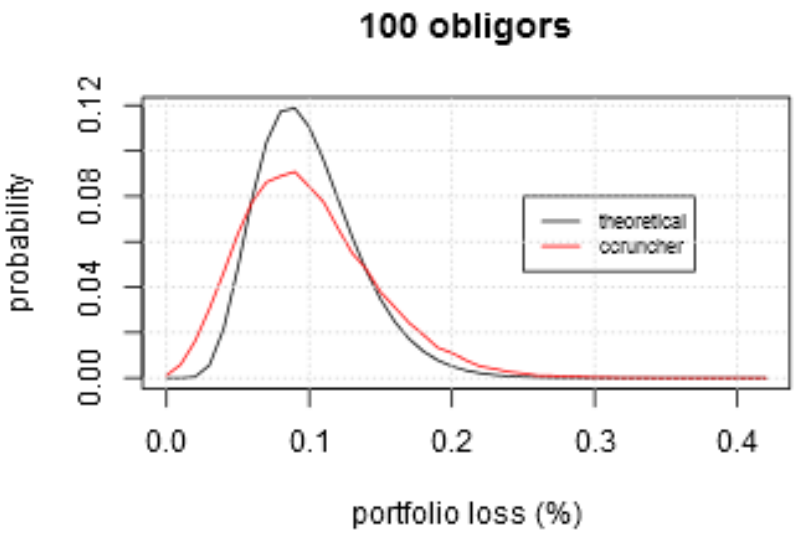
Some Results – LHP Replication

A simple case: The Large Homogenous Pool

1000 obligors (~∞)



100 obligors



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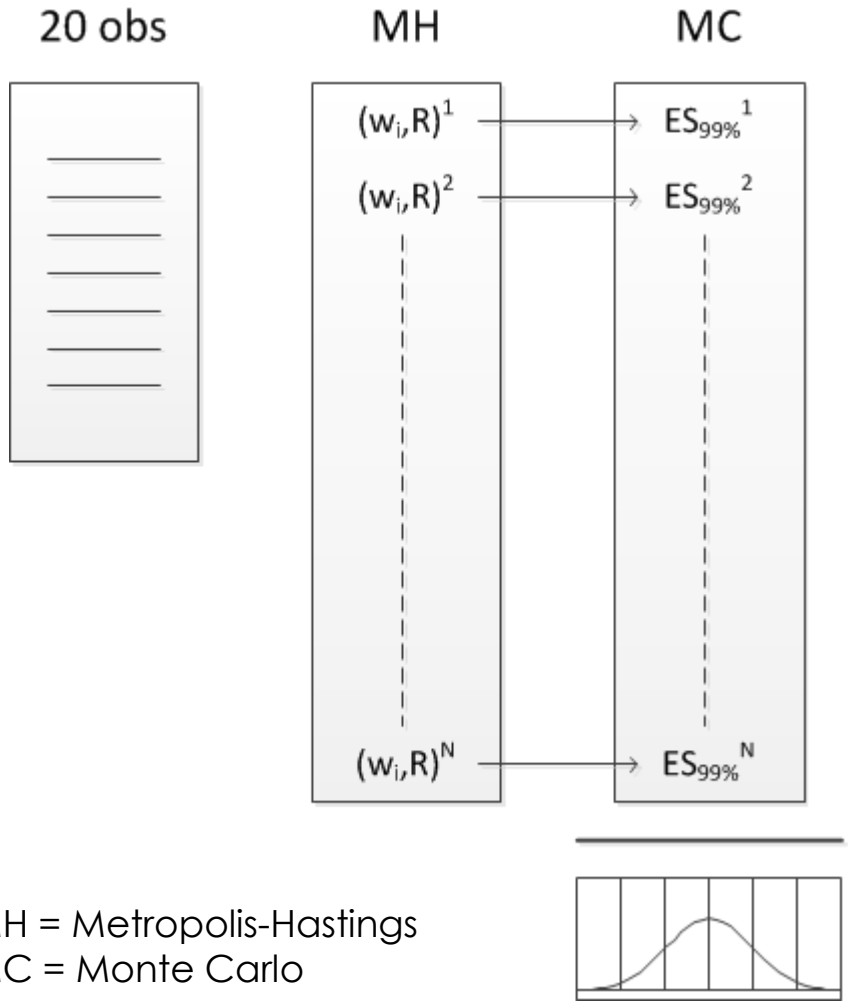
Some Results – Params Uncertainty (I)



MH = Metropolis-Hastings
MC = Monte Carlo

Credit Risk estimator (eg. $ES_{99\%}$) can be considered a distribution instead of a fixed value.

The stochastic behavior is due to parameter uncertainty.

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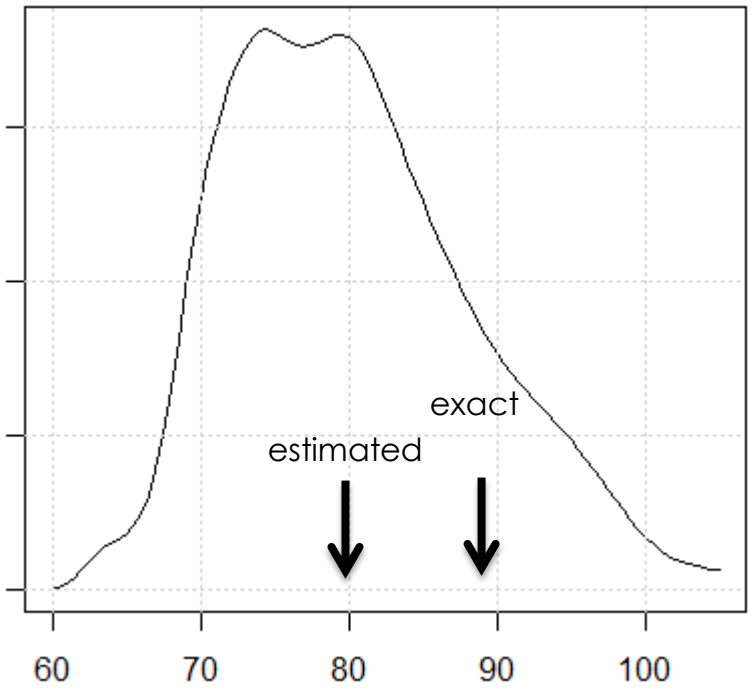
Some Results – Params Uncertainty (II)

Expected Shortfall 99%



N = 155

exact = parameters used to simulate observations

estimated = parameters estimated from 20 observations

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Some Results – Others

Download the software from http://ccruncher.net
Use it to analyze your portfolio !

- Define your risk statistics
- How many factors ?
- Exposures/Recoveries impact
- Risk disaggregation
- Stress-Testing
- …

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Conclusions

CURRENT

- Exact parameters
- Simplified model
- Basic computation

PROPOSAL

- Fuzzy parameters
- Accurate model
- Intensive computation

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