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Mathematical modelling of heat exchange in flash tank heat exchanger cascades

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MATHEMATICAL MODELLING OF HEAT EXCHANGE IN FLASH TANK HEAT EXCHANGER CASCADES

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ABSTRACT. Flash tank evaporation combined with a condensing heat exchanger can be used when heat exchange is required between two streams, where at least one of these streams is difficult to handle (tends severely to scale, foul, causing blockages). To increase the efficiency of heat exchange, a cascade of these units in series can be used. Heat transfer relationships in such a cascade are very complex due to their interconnectivity; thus the impact of any changes proposed is difficult to predict. Moreover, the distribution of loads and driving forces at different stages and the number of designed stages faces tradeoffs which require fundamental understanding and balances. This paper addresses these problems, offering a mathematical model of a single unit composed of a flash tank evaporator combined with a condensing heat exchanger. This model is then extended for a chain of such units. The purpose of this model is an accurate study of the factors influencing efficiency of the system (maximizing heat recovery) and evaluation of the impact of an alteration of the system, thus allowing for guided design of new or redesign of existing systems.

LIST OF NOTATIONS

A	Heat exchanger area, m^2;
c_p	Specific heat capacity, J/(kg.K);
F	Hot stream mass flow rate, kg/s;
G	Steam flow rate, kg/s;
h	Specific enthalpy, J/kg;
h_{fg}	Latent heat, J/kg;
L	Output hot stream mass flow rate, kg/s;
P	Pressure, Pa;
Q	Power, W;
S	Cold stream flow rate, kg/s;
T	Slurry (hot stream) temperature, degC;
t	Spent liquor (cold stream)temperature, degC;

1

t_C	Condensation temperature, degC;
t_G	Steam temperature, degC;
U	Heat transfer coefficient, W/(m^2.K);
ϕ	Temperature change due to non condensable gases, K;
ψ	Temperature change due to superheating, K;

Subscripts and superscripts

F	Input hot stream;
G	Steam flow;
N	Number of flash tank stages;
L	Output hot stream;
S	Cold stream;
Δt_{LM}	Log Mean Temperature Difference;
W	Water;
in	Input;
out	Output;
$*$	Equilibrium value.

1. Introduction

Effective heat exchange between hot and cold streams is an important part of energy efficiency in process operations. In the real-life industrial practice, a situation, where one or both of these streams are difficult to handle in a standard shell and tube heat exchanger due to the nature of the liquids (chemical aggressiveness, or a tendency to scale or foul because of solid particles content or high salt content, etc.), is rather common. In these cases, an effective heat exchange can be organized using a flash tank evaporation combined with a condensing heat exchanger. In this system, vapour is generated from the hot stream through pressure drop in the flash tank and then condenses in the heat exchanger, transferring the latent heat to the cold stream. Evaporation due to pressure drop in the flash tank is cooling the hot stream; this is followed by condensation (and the release of the latent heat of evaporation) in the condenser, which is used as a heat exchanger to heat the cold stream. In this manner the water vapour is used as the heat transferring agent from one stream to the other, substituting the indirect recuperative heat transfer. In some cases the vapour can be directly injected into the cold stream; this remarkably increases the efficiency of the heat exchange. However, the direct injection cannot be used in all cases, as there often is a need to remove an excess of water from the liquid.

Often to maximize the heat exchange, a train of these units arranged in a sequence is used, with the hot stream flowing through the flash tanks and the cold stream flowing in a counter current direction through the heat exchangers (the shell and tube type of condensers). The interlinked nature of the flash tanks and the condensing heat exchangers in series makes analysis of the heat exchange

process difficult, as any alteration in the operating parameters at any point in the chain impacts the other operating conditions in the rest of the train. In such a system, any change of the system parameters could have a difficult to predict impact on the process and the overall efficiency and balance of the heat exchange across the train. While flash tank heat exchange, as described above, is in use in many real-life industrial applications, the particular case study for this project is the system that is commonly in use in alumina production.

Alumina is produced from bauxite through the so-called Bayer process [1, 2, 3, 4], which consists of four main steps (see Fig. 1):

(1) Dissolution of bauxite in an aqueous Sodium hydroxide (NaOH) solution ("Digestion");
(2) Removal of the insoluble materials ("Clarification");
(3) Crystallisation of gibbsite from the liquor ("Precipitation");
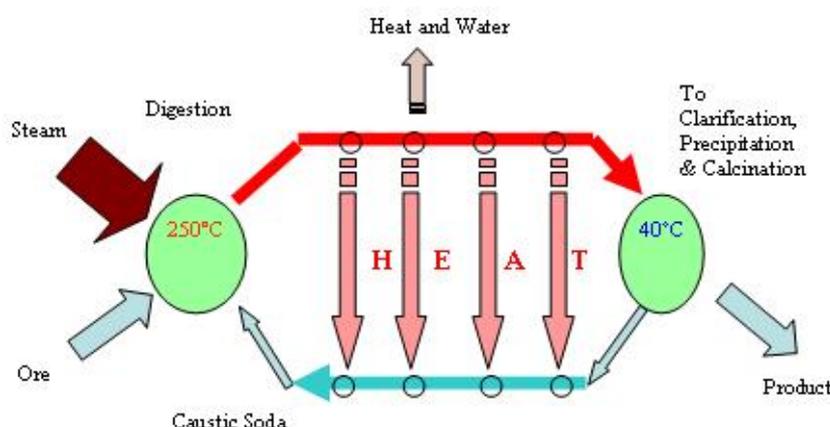(4) Removal of chemically bound water ("Calcination").

FIGURE 1. Basic thermal flow chart of the Digester-Precipitator area for the Bayer process.

The Bayer process uses the fact that at a high temperature alumina is dissolvable in caustic soda. In the digestion area the bauxite ore is mixed with caustic soda and the reheated spent liquor (the recycled caustic soda), and the mixture is heated to 250 °C by adding high pressure steam; at this stage alumina is dissolved in caustic soda. The resulting hot slurry (a mixture of the solution of alumina in caustic soda, the insolvable minerals and hot water) then passes through a sequence of flash tanks, where the mix is cooled through indirect heat exchange with a cold liquid stream of the spent liquor. The cooling of the "cold fill" slurry to a temperature of 40 °C is essential for the next stages in the alumina production process (namely, crystallization of the dissolved alumina, or "precipitation"). After removing of the insoluble minerals and the crystallized alumina from the

mix, the remaining caustic soda ("the spent liquor") is returned into the digester for further use. It is obvious that by utilizing the heat that is removed from the hot stream in the flash tank train for heating the cold spent liquor, the amount of energy (in the form of high pressure steam) required in the digester can be considerably reduced.

Regarding the very energy intensive character of the Bayer process, many developments, aiming at an energy demand reduction, were suggested. Thus it was suggested to re-use the low-grade heat of calcination for spent liquor evaporation [5], or to introduce a new type Spiral Heat Exchangers for the digester liquor cycle internal heat exchange improvement [6]. Both these approaches appear to be reasonable. On one hand, by increasing the amount of heat transferred in the flash tank train from the hot stream leaving the digester to the cold spent liquor, the amount of high pressure steam required in the digester will be reduced, resulting in an increase in the overall efficiency of the process. On the other hand, the low-grade heat recovery and reuse, where it is possible in the whole technological process, can partly replace and reduce the demand for the high potential heat, which is currently supplied by the hot stream digester line. In both cases, however, an appropriate mathematical model is needed in order to optimize the internal and external heat streams. The objective of this work is to construct a basic model of a flash tank and condenser chain that would allow a reasonably accurate evaluation of the impact of any structural of parametrical alteration of the system, thus allowing for guided design of new, or redesign of existing systems.

2. Model of a unit composed of a flash tank and a condenser

A good level of understanding of underlying physical mechanisms of the process is essential for a correct model composition. Fig. 2 shows the principal flow chart of a flash tank – condenser unit and the main physical parameters; Fig. 3 represents the process state diagram with respect to the pressure-temperature coordinate system. Here, the hot slurry enters the flash tank at flow rate F and with specific heat c_{pF}, pressure P_0, temperature T_F, and enthalpy h_F. The essential condition for the successfully operation of the system is that in the flash tank the pressure drops from P_0 to $P_1 < P_0$. Another basic assumption is that the input liquid is saturated; that is, it is at point A in the state diagram in Fig. 3. The third assumption is that the pressure in the flash tank and that in the condenser are equal. Due to the pressure drop in the flash tank, water in the slurry partially evaporates, and the vapour with specific heat c_{pG}, pressure P_1, temperature t_G and enthalpy h_G flows at a rate G to the condenser (heat exchanger). The removal of the latent heat with the formed vapour causes drop of the temperature in the flash tank from T_F to $T_L < T_F$. The liquid having pressure P_1, temperature T_L and enthalpy h_L is leaving the flash tank at flow rate L. In the condencer/heat exchanger steam condenses at temperature t_c and pressure P_1,

and exchanges heat Q with the cold liquid, which is entering the exchanger at flow rate S with temperature t_{in} and specific heat c_{ps}. The liquid leaves the heat exchanger at temperature t_{out}.
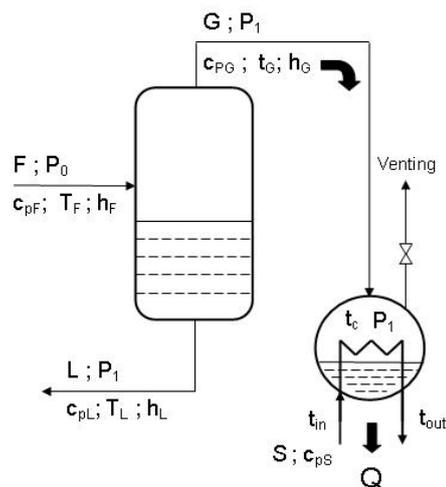

FIGURE 2. A schematic representation of a flash tank – condenser unit and the corresponding physical parameters.

The industrial process has a number of particular properties, which are essential for the modelling; the state diagram in Fig. 3 helps to explain these features.

(1) *The superheated state of the vapour.* The state diagram in Fig. 3 shows two equilibrium curves: curve p_F^* is for of the slurry boiling in the flash tank, and curve p_W^* is for the pure water. As it was already mentioned, the flash tank – condenser heat exchanger systems are preferable used when the hot stream liquid is difficult to treat; usually it is a liquid containing many dissolved (usually non-organic) components, which are disposed to scaling. Boiling temperatures of such liquids (such as the slurry in alumina production) are higher than that of the pure water. The flash evaporation is an equilibrium process, and hence in the flash tank the liquid and the vapour have the same temperature. The expansion from pressure P_0 to P_1 corresponds to the moving along the equilibrium curve p_F^* for the slurry from point A to point D in the state diagram (Fig. 3). However, the vapour is practically pure water vapour, and hence its condensation temperature is at point C on the pure water equilibrium curve p_W^*. Therefore, the vapour is superheated by ψ degrees (Fig. 3), and hence to begin the condensation in the condenser, the vapour firstly must be cooled to the temperature t_c.
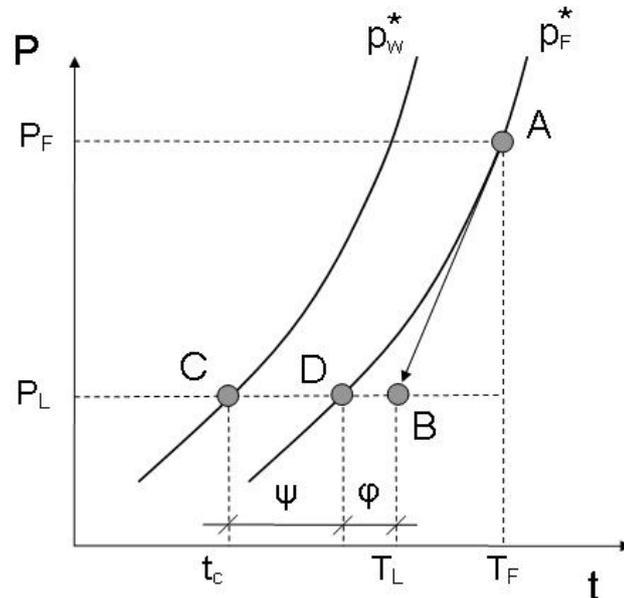
FIGURE 3. State diagram for heat exchange in a unit.

(2) *The influence of non-condensable gases (NCG).* Real-life industrial liquids always contain some dissolved non-condensable gases (NCG), which are released by the evaporation. As a result, the partial pressure of the water vapour in the flash tank is lower than the overall pressure P_1, and respectively the condensation begins at a lower temperature. This is represented in Fig. 3 as a real flashing process from point A to point B (instead of point D). The impact is equivalent to an additional superheating by φ degrees.

(3) *The condensation occurs at constant temperature.* Consequently, for a single stage the actual arrangement of the flows in the condenser (co-current or counter-current) is of no importance. This can be considered as an advantage because it implies that there are no special requirements to the heat exchanger design; however, a general disadvantage of this is that the temperature driving force decreases (there is a loss of potential).

(4) *Main physical comments.* In the flash tank - condenser unit the heat transfer consists in two sequent stages of very different character: (1) simultaneous heat and mass transfer with direct contact between the phases; and (2) indirect heat exchange with the external stream trough the wall in the condenser. At the first stage the heat is transferred from the evaporation in the flash tank to the condensation in the condenser as the latent heat of the carrier agent — the water vapour. The movement of the agent is only one-way from the flash tank to the condenser (unlike to that in the

heat pipes), where the condensate must be constantly removed from. The NCG content dramatically deteriorates the condensation: as it was shown in [7], a presence of a bulk mass fraction of air equal to just 0.5% leads to a reduction of 50% or more in heat transfer. Therefore, a venting line for maintaining NCG at a low level is essential for efficient heat transfer. At the second stage, ordinary indirect heat exchange is going on.

For mathematical modelling of each stage, the process is assumed continuous, with stationary regimes and the well defined zones (e.g. flash tank, condenser) with concentrated parameters. For the sake of simplicity, integrated forms of the equations can be used, which leads to a relatively simple system of algebraic equations.

The model of the unit composed of a flash tank and a condenser is based on equality of the heat generated by the flash tank (the source) and the heat consumed in the condenser (the sink). The mass balance in the flash tank (Fig. 2) is given by the equality

$$(2.1) \qquad\qquad F = L + G,$$

and the enthalpy balance in the tank is

$$(2.2) \qquad\qquad Fh_F = Lh_L + Gh_G.$$

Solving these equations for L and G yields

$$(2.3) \qquad\qquad L = F\frac{h_G - h_F}{h_G - h_L}$$

$$(2.4) \qquad\qquad G = F\frac{h_F - h_L}{h_G - h_L}$$

The generated heat power is equal to the latent heat of the water evaporated in the flash tank (which is the carrier of the heat flow):

$$(2.5) \qquad\qquad Q = Gh_{fg}.$$

Taking into account that, by definition, $h_G = h_L + h_{fg}$, $h_F = c_{pF}T_F$, $h_L = c_{pL}T_L$, assuming that $c_{pF} \approx c_{pL}$,

$$(2.6) \qquad\qquad L = \frac{F}{h_{fg}}\left[h_{fg} - c_{pF}\left(T_F - T_L\right)\right]$$

$$(2.7) \qquad\qquad G = \frac{Fc_{pF}\left(T_F - T_L\right)}{h_{fg}}$$

we obtain the following expression for the generated heat power:

$$(2.8) \qquad\qquad Q = Gh_{fg} = Fc_{pF}(T_F - T_L).$$

Since $h_{fg} \gg c_{pF} \left(> T_F - T_L\right)$ we use the approximation $F = L$ in our calculations.

From the Fig. 3 it can be seen that $T_L = t_c + \varphi + \psi$ (Point B). As was mentioned above, here ψ expresses the superheated state of the vapours when the evaporating liquid in the flash tank contains dissolved non-organic components,

and φ is the temperature correction due to the presence of non-condensable gases in the vapour. Substituting this definition of T_L into (2.8) yields

(2.9) $$Q = Fc_{pF}(T_F - t_c - \varphi - \psi).$$

Consumed heat in the condenser (the sink) can be expressed by the main integrated heat exchange equation for indirect heat transfer (that is valid for both counter-current and co-current streams) as:

(2.10) $$Q = UA\Delta t_{LM},$$

where

(2.11) $$\Delta t_{LM} = \frac{(t_c - t_{in}) - (t_c - t_{out})}{\ln\frac{t_c - t_{in}}{t_c - t_{out}}} = \frac{t_{out} - t_{in}}{\ln\frac{t_c - t_{in}}{t_c - t_{out}}}$$

is the mean logarithmic temperature difference expressed as a function of the temperature differences on both ends of the heat exchanger. Furthermore, from the heat balance of the cold external stream it follows that

(2.12) $$Q = Sc_{pS}(t_{out} - t_{in}).$$

Combining equations (2.10–2.12) and excluding t_c and t_{out}, we obtain the heat power exchanged between the hot and cold streams by the flash tank – condenser unit:

(2.13) $$Q = \frac{K_S \frac{C-1}{C}}{1 + \frac{K_S(C-1)}{K_F C}}(T_F - t_{in} - \psi - \varphi).$$

Here $K_F = Fc_{pF}$, $K_S = Sc_{pS}$ and

(2.14) $$C = \exp\left(\frac{UA}{K_S}\right).$$

It is noteworthy, that equation (2.13) does not include the pressures and the condensation temperature, and hence is very convenient for the analysis of the system flash tank – condenser. Firstly, it can be seen that for the given flow rates and temperatures of the streams, the exchanged heat power depends only on heat transfer coefficient U and the heat exchange area of the condenser A. Secondly, for given heat exchanger parameters and stream flow rates, Q depends on the hot and cold streams input temperatures with the obvious condition

(2.15) $$T_F > t_{in} + \varphi + \psi.$$

Furthermore, in Fig. 2 temperature T_F corresponds to point A on the equilibrium line of the hot slurry p_F^*, and hence it depends on the pressure of the input hot stream. Therefore, it follows from inequality (2.15) that the only way to ensure satisfactory efficiency of the heat exchange is to ensure sufficiently high pressure.

The relationship between the equilibrium values of the pressure and temperature is given with a satisfactory accuracy by the Antoine equation

$$(2.16) \qquad \log p^* = K_1 - \frac{K_2}{K_3 + t},$$

where K_1, K_2 and K_3 are constants, in a wide temperature range.

3. CASCADE OF THE UNITS

In order to increase the efficiency of heat exchange, the flash tank - condenser units are often assembled in sequential chain. The schematic representation of a series of flash tanks and heat exchangers are shown in Fig. 4. The top line represents the flow of hot slurry from the digester D. It starts at a temperature T_1, and goes through a series of N flash tanks. At the ith flash tank, the slurry's temperature decreases from T_i at the input to T_{i+1} at output (which is the input temperature for the $(i+1)$th flash tank), and the slurry finally exits the cascade at temperature $T_{N+1} = T_{\text{out}}$. The bottom line represents the flow of spent liquor into the digester. The flow starts at temperature $t_{N+1} = t_{\text{in}}$, and as it passes through the ith condenser, its temperature increases from t_{i+1} to t_i. The spent liquor enters the digester at temperature t_1. The closer this temperature is to the temperature at which the digester operates (250°C) the less energy is needed to heat it up.
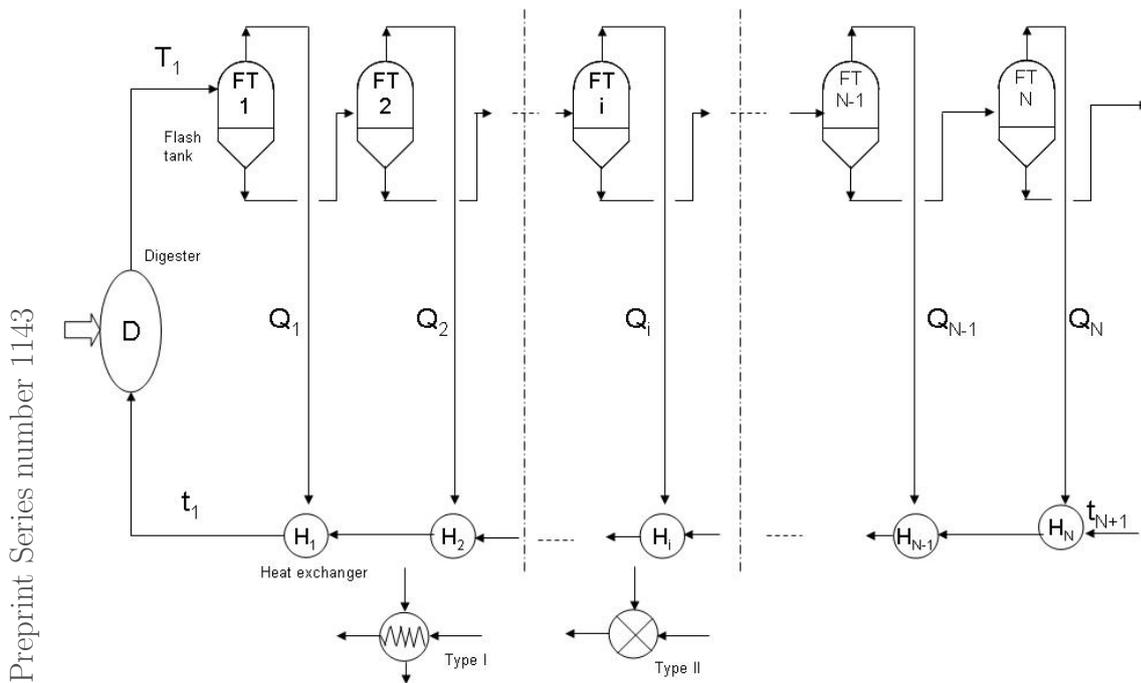
FIGURE 4. A cascade of the flash tank - condenser units

The analysis of a single condenser in Section 2 shows that the four important temperatures, T_i, T_{i+1}, t_i and t_{i+1}, are related by two (approximately) linear equations. In this paper, we assume that these equations are precisely linear. Then for the $2N + 2$ variables $T_1, \ldots, T_{N+1}$ and $t_1, \ldots, t_{N+1}$ we have $2N$ linear equations. In addition, we have fixed boundary conditions, namely $T_1 = 250°C$, which is the temperature at which the digester operates. This leaves one free variable, which we shall choose to be $t_{N+1} = t_{\text{in}}$. In this case following the general theory of linear systems, all the remaining variables are then linearly functions of t_{in} and T_1. Thus, we have

$$(3.1) \qquad t_1 \;=\; a\, t_{\text{in}} + b$$

where a is a real number, which from physical principles must be between 0 and 1.

From Section 2, we get that the amount of heat exchanged in the ith unit is given by

$$(3.2) \qquad Q_i \;=\; D_i(T_i - t_{i+1} - \pi_i),$$

where D_i is a constant depending on the physical properties of the unit,

$$(3.3) \qquad D_i \;=\; \left(\frac{K_S \frac{C-1}{C}}{1 + \frac{K_S(C-1)}{K_F C}} \right)_i ,$$

and $\pi_i = \psi_i + \phi_i$ is a loss in driving force potential (temperature difference) caused by superheating (the ψ_i term) and the presence of non-condensable gases (the ϕ_i term). Let the flow rate of the spent liquor be S_i and its specific heat be c_{pi} (these may change from stage to stage as the amount of water in the spent liquor changes). Let the flow rate of the slurry be F_i and its specific heat c_{pFi} (again, allowing for variation of these quantities from stage to stage). Then we have

$$(T_i - T_{i+1})F_i c_{pFi} = Q_i,$$
$$(t_i - t_{i+1})S_i c_{pSi} = Q_i.$$

Let

$$K_i = D_i / F_i c_{pFi},$$
$$k_i = D_i / S_i c_{pS}i.$$

Using 3.2, we get the equations

$$(3.4) \qquad T_i - T_{i+1} = K_i(T_i - t_{i+1} - \pi_i),$$
$$(3.5) \qquad t_i - t_{i+1} = k_i(T_i - t_{i+1} - \pi_i).$$

As i ranges from 1 to N, equations (3.4) and (3.5) give $2N$ linear equations. If we augment them with the two equations

$$(3.6) \qquad T_1 = T_{in},$$
$$(3.7) \qquad t_{N+1} = t_{\text{in}},$$

we get a system of $2N + 2$ linear equations in the $2N + 2$ variables $T_1, \ldots, T_{N+1}$ and $t_1, \ldots, t_{N+1}$. They can be written in matrix form as $A\vec{t} = \vec{b}$, where, using a superscript t to denote the transpose,

$$\vec{t} = (t_1, T_1, t_2, T_2, \ldots, t_{N+1}, T_{N+1})^t,$$

$$\vec{b} = (-k_1\pi_1, -K_1\pi_1, -k_2\pi_2, -K_2\pi_2, \ldots, -k_N\pi_N, -K_N\pi_N, 255, t_{\text{in}})^t$$

and A is the coefficient matrix

$$\begin{pmatrix} 1 & -k_1 & k_1-1 & 0 & 0 & 0 & \ldots \\ 0 & 1-K_1 & K_1 & -1 & 0 & 0 & \ldots \\ 0 & 0 & 1 & -k_2 & k_2-1 & 0 & \ldots \\ 0 & 0 & 0 & 1-K_2 & K_2 & -1 & \ldots \\ \vdots & & & & & & \\ 0 & 1 & 0 & 0 & \ldots & & \\ 0 & 0 & \ldots & & & 0 & 1 \end{pmatrix}.$$

The existing plant has an assembly of 10 heat exchangers in a row. This leads to 30 physical parameters that we need to know: $K_1, \ldots, K_{10}$, $k_1, \ldots, k_{10}$ and $\pi_1, \ldots, \pi_{10}$. If π_i is known, we could use equations (3.4) and (3.5) to calculate K_i and k_i.

The following Theorem holds for this model:

Theorem 1. *An assembly of N heat conversion units, where heat exchange in every unit described by the system*

$$T_{i+1} = T_i - Q_i/F_i c_{pFi},$$

$$t_i = t_{i+1} + Q_i/S_i c_{pSi},$$

$$Q_i = D_i(T_i - t_{i+1} - \pi_i),$$

can be described by a model

$$T_{N+1} = (1 - \overline{K})T_1 - \overline{K}t_{N+1} + \overline{K}\,\overline{\delta},$$

$$t_1 = (1 - \overline{k})t_{N+1} + \overline{k}T_1 - \overline{k}\,\overline{\Delta}.$$

Proof. At every stage,

$$T_{i+1} = (1 - K_i)T_i + K_i t_i + K_i \pi_i,$$

$$t_i = (1 - k_i)t_{i+1} + k_i T_i - k_i \pi_i,$$

where $k_i = D_i/S_i c_{pSi}$ and $K_i = D_i/F_i c_{pFi}$, hold. The boundary conditions T_1 and t_{N+1} are assumed to be given. Hence we have $N + 1$ linear algebraic equations for $N + 1$ unknown variables, and the system is well defined.

The hypothesis of Theorem definitely holds for $N = 1$. Assume that this holds for an assembly of $i - 1$ heat exchangers, from the first to the $(i - 1)$st. That is,

we assume that the following equations hold:

$$(3.8) \qquad T_i = (1 - K_{1,i-1})T_1 + K_{1,i-1}t_i + K_{1,i-1}\delta_{1,i-1},$$

$$(3.9) \qquad t_1 = (1 - k_{1,i-1})t_i + k_{1,i-1}T_1 - k_{1,i-1}\Delta_{1,i-1}.$$

We now have to prove that then it also holds for an assembly of i heat exchangers.

For the ith unit the following equations hold:

$$(3.10) \qquad T_{i+1} = (1 - K_i)T_i + K_i t_{i+1} + K_i \pi_i,$$

$$(3.11) \qquad t_i = (1 - k_i)t_{i+1} + k_i T_i - k_i \pi_i.$$

Substituting (3.11) into (3.8), and (3.8) into (3.11), we obtain

$$T_i = (1 - K_{1,i-1})T_1 + K_{1,i-1}[(1 - k_i)t_{i+1} + k_i T_i - k_i \pi_i]t_i + K_{1,i-1}\delta_{1,i-1},$$

$$t_i = (1 - k_i)t_{i+1} + k_i[(1 - K_{1,i-1})T_1 + K_{1,i-1}t_i + K_{1,i-1}\delta_{1,i-1}] - k_i \pi_i,$$

and hence

$$(1 - k_i K_{1,i-1})T_i = (1 - K_{1,i-1})T_1 + K_{1,i-1}(1 - k_i)t_{i+1} - K_{1,i-1}k_i \pi_i + K_{1,i-1}\delta_{1,i-1},$$

$$(1 - k_i K_{1,i-1})t_i = (1 - k_i)t_{i+1} + k_i(1 - K_{1,i-1})T_1 + k_i K_{1,i-1}\delta_{1,i-1} - k_i \pi_i.$$

Substituting these equalities into (3.10) and (3.9), we finally get

$$T_{i+1} = \frac{(1-K_i)}{(1-k_i K_{1,i-1})}\big((1-K_{1,i-1})T_1 + K_{1,i-1}(1 - k_i)t_{i+1} - K_{1,i-1}k_i \pi_i + K_{1,i-1}\delta_{1,i-1}\big)$$
$$+ K_i t_{i+1} + K_i \pi_i \ ,$$

$$t_1 = \frac{(1 - k_{1,i-1})}{(1 - k_i K_{1,i-1})}\big((1 - k_i)t_{i+1} + k_i(1 - K_{1,i-1})T_1 + k_i K_{1,i-1}\delta_{1,i-1} - k_i \pi_i\big)$$
$$+ k_{1,i_1}T_1 - k_{1,i-1}\Delta_{1,i-1}.$$

That is,

$$(3.12) \qquad T_{i+1} = (1 - K_{1,i})T_1 + K_{1,i}t_{i+1} + K_{1,i}\delta_{1,i},$$

$$(3.13) \qquad t_1 = (1 - k_{1,i})t_{i+1} + k_{1,i}T_1 - k_{1,i}\Delta_{1,i},$$

where the coefficients $K_{1,i}, k_{1,i}, \Delta_{1,i}, \delta_{1,i}$ are defined by recurrent formulae

$$(3.14) \qquad k_{1,i} = \frac{k_{1,i-1} + k_i - k_i k_{1,i-1} - k_i K_{1,i-1}}{1 - k_i K_{1,i-1}} = 1 - \frac{(1 - k_i)(1 - k_{1,i-1})}{1 - k_i K_{1,i-1}},$$

$$(3.15) \qquad K_{1,i} = \frac{K_{1,i-1} + K_i - K_i K_{1,i-1} - k_i K_{1,i-1}}{1 - k_i K_{1,i-1}} = 1 - \frac{(1 - K_i)(1 - K_{1,i-1})}{1 - k_i K_{1,i-1}},$$

$$(3.16) \qquad k_{1,i} \cdot \Delta_{1,i} = k_{1,i-1}\Delta_{1,i-1} - (1 - k_{1,i-1})\frac{k_i K_{1,i-1}\delta_{1,i-1} - k_i \pi_i}{1 - k_i K_{1,i-1}},$$

$$(3.17) \qquad K_{1,i} \cdot \delta_{1,i} = K_i \pi_i + (1 - K_i)\frac{K_{1,i-1}\delta_{1,i-1} - k_i K_{1,i-1}\pi_i}{1 - k_i K_{1,i-1}}. \qquad \square$$

Equations (3.12–3.17) enable us to find explicitly the temperatures at the end of a given cascade of heat converters. We have to stress that these equations also hold for any ordered sub-sequence composed of any number of units. Furthermore, for many practically relevant heat exchanges assemblies equations (3.12, 3.13) can be further simplified. To show this we have to recollect firstly that parameter π_i is depends exclusively on the chemical properties of the hot stream, and in many practically relevant cases these properties can be assumed constant throughout all units of an assembly. Indeed, by definition, term π_i is the sum of two components, ψ_i and ϕ_i, and it reflects a loss in driving force caused by superheating (the ψ_i term) and the presence of non-condensable gases (the ϕ_i term) in the hot stream. Superheating occurs because the liquid is a mixture of chemicals hence vaporization stars at a temperature higher that for pure water. We estimate that ψ_i is approximately 6.5 K. The term ϕ_i is trickier to estimate, but should be less than 1 K. We assume that

$$(3.18) \qquad \pi_i \approx 7.5.$$

It is apparent, however, that the value of π does not change significantly from unit to unit and is near constant throughout the sequence of heat exchanging units. For such a case, that is if $\pi_i = \pi$ for all i,

$$(3.19) \qquad \Delta_{1,i} = \delta_{1,i} = \pi$$

hold, and the equations (3.12, 3.13) take the form

$$(3.20) \qquad T_{i+1} = (1 - K_{1,i})T_1 + K_{1,i}t_{i+1} + K_{1,i}\pi,$$

$$(3.21) \qquad t_1 = (1 - k_{1,i})t_{i+1} + k_{1,i}T_1 - k_{1,i}\pi.$$

It is obvious that (3.19) hold for a single unit. For two units such that $\pi_1 = \pi_2 = \pi$ hold, equations (3.14–3.17) give

$$k_{1,2} \cdot \Delta_{1,2} = \left(k_1 - (1 - k_1)\frac{k_2 K_1 - k_2}{1 - k_2 K_1} \right)\pi = k_{1,2}\pi,$$

$$K_{1,2} \cdot \delta_{1,2} = \left(K_2 + (1 - K_2)\frac{K_1 - k_2 K_1}{1 - k_2 K_1} \right)\pi = K_{1,2}\pi.$$

This recurrent process can be carried further on.

4. Conclusion

In this paper we considered the heat recovery in a complex cascade of the flash tank – condenser heat exchange units. We start at constructing a simple basic model of a single unit composed of a flash tank and a condenser/heat exchanger; the simplicity of this model enables us to extend it to a chain of such units, and to obtain explicit recurrent formula for the heat exchange in such a chain.

There is a number of practically important conclusions that immediately follow from this analysis. An important conclusion that follows from equation (2.8)

is that the system flash tank – condenser (a single unit) is self-regulated: the consumer (the sink) determines the heat generation in the flash tank. In industrial applications the condenser is often not the only consumer, as a fraction of the steam can be spited from the flash tank for other external consumers. It is remarkable that in such a case the flash tank generates more vapour, but this requires a higher pressure drop in the flash tank. Such a complication does not change the principal postulates of the model and can be straightforwardly accounted with a very small change in the model by introducing a splitting coefficient. This, however, was out of the scop of the present paper. We would like also to mention another possible complication that can be immediately incorporated into the model, namely a possibility for an introducing mixing condensers in the cold line of the cascade (that is, direct injection of the vapour into the cold stream, as it is show as an option in Fig. 4).

Another important and practically relevant conclusion, that readily follows from equations (2.8) and (2.9), is that for the basic model without additional export of vapour with the same condensers in all units the temperature distribution in the cascade is uniform. This conclusion was confirmed by numerical experiments [8]. It is noteworthy, however, that by (2.10) the pressure is not evenly distributed.

The model includes the influence of the NCG and the superheated state of the vapour generated in the flash tank. Both effects lead to decrease of the efficiency and can be taken into account, apart from the temperature increments ψ and φ, by recalculating the heat exchange coefficient U in equation (2.10). Superheated vapour must be cooled to the temperature of condensation before the condensation can start. The gas cooling is a very slow process; the heat transfer coefficient for the gas cooling is more than 50 times less than that of condensation. This implies that a part of a condenser area is used for the gas cooling instead of condensation, thus decreasing the effective area of condensation. In the case when the overall heat exchange area is used modelling (which is common in engineering practice), then this reduction can be accounted for either by introducing an idea of effective heat exchange area, or by reducing the effective heat exchange coefficient (cf. (2.10)). The impacts of these two factors are very important, as, for instance, in the alumina industry the usual value of the sum $\psi + \varphi$ is of about 6.5 degrees; this leads to approximately 11% of reduction of U [8]. The model proposed can be used to optimize the heat recovery in the flash tank heat exchanger cascades in general, and for the Bayer process digester section specifically. Obvious further steps are to enhance the model taking into account heat export from the flash tank system and software development to enable simulation of different scenarios for efficiency improvement of the entire heat recovery system.

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