On the plane strain of thermo-microstretch elastic solids

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Abstract

This paper is concerned with the linear theory of thermo-microstretch elastic solids. We present a method to reduce the thermoelastic plane strain problem to an isothermal one with zero body loads and with certain boundary data. The result is used to study the thermoelastic deformation of a tube and the problem of thermal stresses in a cylinder subjected to a uniform temperature gradient.

1. Introduction

In a series of papers Eringen [1-4] established the general theory of micromorphic continua for prediction of behavior of materials with inner structure. In [3], Eringen introduced the theory of microstretch elastic solids. This theory is a generalization of the theory of micropolar elasticity. The material points of microstretch solids can stretch and contract independently of their translations and rotations. The microstretch continua are used to model composite materials reinforced with chopped fibers and various porous media. In [4], Eringen has extended the theory of microstretch elastic solids to include the heat conduction. The linear theory of thermoelastic microstretch solids has been studied in various papers (see, e.g., [5], [6]).

In this paper we consider the equilibrium theory of thermo-microstretch elastic solids. In the framework of the classical thermoelasticity, Muskhelishvili [7] established a method of reduction of the thermoelastic plane problem to an isothermal one. In what follows we generalize the method of Muskhelishvili to the plane strain problem for thermo-microstretch elastic

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bodies. The result is used to study the thermoelastic deformation of a tube and the problem of thermal stresses in a cylinder subjected to a uniform temperature gradient.

2. Basic Equations

In this section we summarize the basic equations of the linear theory of isotropic microstretch thermoelastic solids. We let B denote the bounded regular region of the three-dimensional Euclidean space occupied by the body at time t_0. The deformation of the body is referred to a system of rectangular Cartesian axes Ox_i and to the reference configuration B. Throughout this paper Latin indices have the range $1, 2, 3$, Greek indices have the range $1, 2$, and the usual summation convention is employed. We call ∂B the boundary of B, and designate by n_i the outward unit normal of ∂B.

Let u_i be the displacement field, and let φ_i be the microrotation field on B. Further, let ψ be the microstretch function. We define the linear strain measures e_{ij}, κ_{ij} and γ_i by [2]

$$e_{ij} = u_{j,i} + \varepsilon_{jik}\varphi_k, \quad \kappa_{ij} = \varphi_{j,i}, \quad \gamma_i = \psi_{,i}, \tag{2.1}$$

where ε_{ijk} is the alternating symbol. In these equations $f_{,i}$ denotes partial differentiation of f with respect to the reference coordinate x_i.

We designate by t_{ij} the stress tensor and by m_{ij} the couple stress tensor. Let π_i be the microstretch vector and let q_i be the heat flux vector. The surface traction, the surface couple, the microstretch traction, and the heat flux at regular points of ∂B are given by

$$t_i = t_{ji}n_j, \quad m_i = m_{ji}n_j, \quad \tau = \pi_i n_i, \quad q = q_i n_i, \tag{2.2}$$

respectively.

The equilibrium equations in the absence of body loads can be written as

$$t_{ji,j} = 0, \quad m_{ji,j} + \varepsilon_{irs}t_{rs} = 0, \quad \pi_{i,i} - \sigma = 0, \tag{2.3}$$

where σ is the microstretch net pressure [4]. In the framework of the equilibrium theory, the energy equation, in the absence of heat sources, reduces to

$$q_{i,i} = 0. \tag{2.4}$$

Let T be the temperature measured from the constant reference temperature T_0. The constitutive equations of homogeneous and isotropic microstretch thermoelastic solids are given by

$$\begin{aligned}
&t_{ij} = (\lambda e_{rr} + \lambda_0\psi - \beta_0 T)\delta_{ij} + (\mu+\kappa)e_{ij} + \mu e_{ji},\\
&m_{ij} = \alpha\kappa_{rr}\delta_{ij} + \beta\kappa_{ji} + \gamma\kappa_{ij} + b_0\varepsilon_{kji}\gamma_k,\\
&\pi_i = a_0\gamma_i + b_0\varepsilon_{irs}\varphi_{r,s},\\
&\sigma = \lambda_0 e_{rr} + \lambda_1\psi - \beta_1 T,\\
&q_i = kT_{,i},
\end{aligned} \tag{2.5}$$

where δ_{ij} is Kronecker's delta and the constitutive coefficients are constants. We assume that the internal energy density is a positive definite quadratic form. This assumption implies that [4]

$$\begin{array}{lll}
3\lambda + 2\mu + \kappa > 3\lambda_0^2/\lambda_1, & 2\mu + \kappa > 0, & \kappa > 0,\\
3\alpha + \beta + \gamma > 0, & \gamma + \beta > 0, & \gamma - \beta > 0,\\
a_0 > 0, & \lambda_1 > 0. &
\end{array} \tag{2.6}$$

We also suppose that k is strictly positive.

The basic equations of thermoelastostatics of microstretch solids consist of the equilibrium equations (2.3), the energy equation (2.4), the constitutive equations (2.5) and the geometrical equations (2.1) on B. To these equations we must adjoin boundary conditions. In the case of Neumann problem the boundary conditions are

$$t_{ji}n_j = \widetilde{t}_i, \quad m_{ji}n_j = \widetilde{m}_i, \quad \pi_i n_i = \widetilde{\tau}, \quad q_i n_i = \widetilde{q} \text{ on } \partial B, \tag{2.7}$$

where $\widetilde{t}_i, \widetilde{m}_i, \widetilde{\tau}$ and $\widetilde{q}$ are prescribed functions.

3. The plane strain

We assume that the region B from here on refers to the interior of a right cylinder with the open cross-section Σ and the lateral boundary Π. The rectangular Cartesian coordinate frame is supposed to be chosen in such a way that the x_3 - axis is parallel to generators of B. We denote by L the boundary of Σ. We consider the thermoelastic plane strain parallel to the x_1, x_2-plane, characterized by

$$\begin{gathered}
u_\alpha = u_\alpha(x_1,x_2), \quad u_3 = 0, \quad \varphi_\alpha = 0, \quad \varphi_3 = \varphi(x_1,x_2),\\
\psi = \psi(x_1,x_2), \quad T = T(x_1,x_2), \quad (x_1,x_2) \in \Sigma.
\end{gathered} \tag{3.1}$$

The above restrictions, in conjunction with the geometrical equations (2.1) and the constitutive equations (2.5) imply that $e_{ij}, \kappa_{ij}, \gamma_i, t_{ij}, m_{ij}, \pi_i, \sigma$ and q_i are all independent of x_3. The non-zero strain measures are given by

$$e_{\alpha\beta} = u_{\beta,\alpha} + \varepsilon_{\beta\alpha 3}\varphi, \quad \kappa_{\alpha 3} = \varphi_{,\alpha}, \quad \gamma_\alpha = \psi_{,\alpha}. \tag{3.2}$$

The constitutive equations (2.5) show that the non-zero dependent constitutive variables are $t_{\alpha\beta}, t_{33}, m_{\alpha 3}, m_{3\alpha}, \pi_\alpha, \sigma$ and q_α. Further

$$\begin{aligned}
&t_{\alpha\nu} = (\lambda e_{\rho\rho} + \lambda_0\psi - \beta_0 T)\delta_{\alpha\nu} + (\mu+\kappa)e_{\alpha\nu} + \mu e_{\nu\alpha},\\
&m_{\alpha 3} = \gamma\kappa_{\alpha 3} + b_0\varepsilon_{3\alpha\nu}\psi_{,\nu},\\
&\pi_\alpha = a_0\psi_{,\alpha} + b_0\varepsilon_{3\rho\alpha}\varphi_{,\rho},\\
&\sigma = \lambda_0 e_{\rho\rho} + \lambda_1\psi - \beta_1 T,\\
&q_\alpha = kT_{,\alpha}.
\end{aligned} \tag{3.3}$$

The equations of equilibrium (2.3) reduce to

$$t_{\beta\alpha,\beta} = 0, \quad m_{\alpha 3,\alpha} + \varepsilon_{3\rho\nu}t_{\rho\nu} = 0, \quad \pi_{\alpha,\alpha} - \sigma = 0 \text{ on } \Sigma. \tag{3.4}$$

In view of (3.3) the energy equation (2.4) becomes

$$\Delta T = 0 \text{ on } \Sigma, \tag{3.5}$$

where Δ is the Laplacian. Given on Π the surface traction $\widetilde{\mathbf{t}}$, the surface couple $\widetilde{\mathbf{m}}$, the microstretch traction $\widetilde{\tau}$ and the heat flux $\widetilde{q}$, with $\widetilde{\mathbf{t}}, \widetilde{\mathbf{m}}, \widetilde{\tau}$ and $\widetilde{q}$ independent of x_3 and $\widetilde{t}_3 = 0, \widetilde{m}_\alpha = 0$, the boundary conditions (2.7) imply that

$$t_{\beta\alpha}n_\beta = \widetilde{t}_\alpha, \quad m_{\alpha 3}n_\alpha = \widetilde{m}, \quad \pi_\alpha n_\alpha = \widetilde{\tau}, \quad q_\alpha n_\alpha = \widetilde{q} \text{ on } L. \tag{3.6}$$

We note that the temperature field T is the solution of the boundary value problem (3.5), $(3.6)_4$. Generally, we shall treat the temperature field having already been so determined. The determination of the thermoelastic equilibrium consists in finding of the functions u_α, φ and ψ on Σ which satisfy the equations (3.2)-(3.4) on Σ and the boundary conditions $(3.6)_{1-3}$ on L. It follows from (3.2)-(3.4) that the field equations can be expressed in the form

$$\begin{aligned}
&(\mu+\kappa)\Delta u_\alpha + (\lambda+\mu)u_{\beta,\beta\alpha} + \kappa\varepsilon_{\alpha\beta 3}\varphi_{,\beta} + \lambda_0\psi_{,\alpha} - \beta_0 T_{,\alpha} = 0,\\
&\gamma\Delta\varphi + \kappa\varepsilon_{3\alpha\beta}u_{\beta,\alpha} - 2\kappa\varphi = 0,\\
&a_0\Delta\psi - \lambda_0 u_{\rho,\rho} - \lambda_1\psi + \beta_1 T = 0, \text{ on } \Sigma.
\end{aligned} \tag{3.7}$$

4. Generalization of Muskhelishvili's method

In this section we present a method of reduction of the thermoelastic plane strain problem to an isothermal one with zero body loads and with certain known boundary data.

In the context of the classical thermoelasticity Muskhelishvili [7] established a method of reduction of the thermoelastic plane problem to an isothermal one which is based on the assumption that the heat source is absent. The result has been extended to micropolar thermoelastic bodies in [8]. In what follows we generalize the method of Muskhelishvili to the plane strain problem for thermo-microstretch elastic solids. In the absence of heat sources the function T satisfies the equation (3.5). Since T is harmonic in Σ we can define the analytic function

$$F(z) = T + iP, \tag{4.1}$$

of the complex variable $z = x_1 + ix_2$. We introduce the functions w_α by

$$w_1 + iw_2 = \int F(z)dz. \tag{4.2}$$

Clearly, we have

$$w_{1,1} = w_{2,2} = T, \quad w_{1,2} = -w_{2,1} = -P. \tag{4.3}$$

If the domain Σ is simply-connected then w_α are single-valued functions.

The relations (4.3) can be written in the form

$$w_{\alpha,\beta} = T\delta_{\alpha\beta} + \varepsilon_{3\beta\alpha}P. \tag{4.4}$$

We introduce the notation

$$d = (2\lambda + 2\mu + \kappa)\lambda_1 - 2\lambda_0^2. \tag{4.5}$$

Clearly,

$$3d = 2[\lambda_1(3\lambda + 2\mu + \kappa) - 3\lambda_0^2] + (2\mu + \kappa)\lambda_1.$$

Thus, in view of (2.6) we find that

$$d > 0. \tag{4.6}$$

We define the constants A_1 and A_2 by

$$A_1 = (\beta_0\lambda_1 - \beta_1\lambda_0)/d, \ \ A_2 = [(2\lambda + 2\mu + \kappa)\beta_1 - 2\lambda_0\beta_0]/d. \tag{4.7}$$

We note that

$$(2\lambda + 2\mu + \kappa)A_1 + \lambda_0 A_2 = \beta_0, \;\; 2\lambda_0 A_1 + \lambda_1 A_2 = \beta_1. \tag{4.8}$$

Let us introduce the functions u^0_α, φ^0 and ψ^0 on Σ by

$$u_\alpha = u^0_\alpha + A_1 w_\alpha, \;\; \varphi = \varphi^0 + A_1 P, \;\; \psi = \psi^0 + A_2 T. \tag{4.9}$$

We denote

$$e^0_{\alpha\beta} = u^0_{\beta,\alpha} + \varepsilon_{\beta\alpha 3}\varphi^0, \;\; \kappa^0_{\alpha 3} = \varphi^0_{,\alpha}. \tag{4.10}$$

It follows from (3.2), (4.4), (4.9) and (4.10) that

$$e_{\alpha\beta} = e^0_{\alpha\beta} + A_1 T\delta_{\alpha\beta}, \;\; \kappa_{\alpha 3} = \kappa^0_{\alpha 3} + A_1 P_{,\alpha}. \tag{4.11}$$

We introduce the functions $t^0_{\alpha\nu}, m^0_{\alpha 3}, \pi^0_\alpha$ and σ^0 by

$$\begin{aligned}
&t^0_{\alpha\nu} = (\lambda e^0_{\rho\rho} + \lambda_0\psi^0)\delta_{\alpha\nu} + (\mu + \kappa)e^0_{\alpha\nu} + \mu e^0_{\nu\alpha},\\
&m^0_{\alpha 3} = \gamma\kappa^0_{\alpha 3} + b_0\varepsilon_{3\alpha\nu}\psi^0_{,\nu},\\
&\pi^0_\alpha = a_0\psi^0_{,\alpha} + b_0\varepsilon_{3\rho\alpha}\varphi^0_{,\rho},\\
&\sigma^0 = \lambda_0 e^0_{\rho\rho} + \lambda_1\psi^0.
\end{aligned} \tag{4.12}$$

The equations (4.12) are the constitutive equations in the isothermal theory corresponding to the deformation characterization by the displacements u^0_α, the microrotation φ^0, and the microstretch function ψ^0.

If we take into account the relations (4.7) and

$$T_{,\alpha} = \varepsilon_{\alpha\beta 3}P_{,\beta}, \;\; P_{,\alpha} = \varepsilon_{\beta\alpha 3}T_{,\beta},$$

then from Eqs. (3.3), (4.11) and (4.12) we obtain

$$\begin{aligned}
&t_{\alpha\beta} = t^0_{\alpha\beta},\\
&m_{\alpha 3} = m^0_{\alpha 3} + (\gamma A_1 - b_0 A_2)\varepsilon_{3\rho\alpha}T_{,\rho},\\
&\pi_\alpha = \pi^0_\alpha + (a_0 A_2 - b_0 A_1)T_{,\alpha},\\
&\sigma = \sigma^0.
\end{aligned} \tag{4.13}$$

Upon substituting $t_{\alpha\beta}, m_{\alpha 3}, \pi_\alpha$ and σ given by (4.13) into the equations (3.4) and using (3.5), we obtain the equations

$$t^0_{\beta\alpha,\beta} = 0, \;\; m^0_{\alpha 3,\alpha} + \varepsilon_{3\rho\nu}t^0_{\rho\nu} = 0, \;\; \pi^0_{\alpha,\alpha} - \sigma^0 = 0, \tag{4.14}$$

on Σ. We assume that L is a piecewise smooth curve parameterized by its arc length s. The mechanical boundary conditions (3.6) reduce to

$$
\begin{aligned}
&t^0_{\beta\alpha} n_\beta = \widetilde{t}_\alpha, \quad m^0_{\alpha 3} n_\alpha = \widetilde{m} + \eta_1 \frac{dT}{ds}, \\
&\pi^0_\alpha n_\alpha = \widetilde{\tau} + \eta_2 \frac{\partial T}{\partial n}, \quad \text{on } L,
\end{aligned}
\tag{4.15}
$$

where

$$
\eta_1 = \gamma A_1 - b_0 A_2, \quad \eta_2 = b_0 A_1 - a_0 A_2. \tag{4.16}
$$

We conclude that the thermoelastic plane problem associated to the simply-connected domain Σ is reduced to the plane strain problem of isothermal elastostatics ($T = 0$) characterized by the field equations (4.10), (4.12), (4.14) on Σ, and the boundary conditions (4.15) on L.

Remark 1. Assume that
a) the domain Σ is simply-connected;
b) the heat source is absent;
c) the mechanical loads are absent.

Then the thermoelastic plane strain problem associated to Σ reduces to solving of the isothermal elastic plane strain problem corresponding to zero body loads, zero boundary traction, the surface couple $\eta_1 dT/ds$ and the microstretch traction $\eta_2 \partial T/\partial n$.

Let us consider now a finite multiply-connected domain Σ. We assume that inside Σ there are m contours $L_k (k = 1, 2, \cdots, m)$ which have no common points; the outer contours is denoted by L_{m+1}. Since the temperature is single-valued then the function F has the form

$$
F(z) = \sum_{k=1}^{m} B_k \log(z - z_k) + F_0(z), \tag{4.17}
$$

where z_k is a point inside the contour L_k, B_k are real constants and F_0 is analytic and single-valued function on Σ. From (4.2) and (4.17) we obtain

$$
w_1 + i w_2 = z \sum_{k=1}^{m} B_k \log(z - z_k) + \sum_{k=1}^{m} (a_k + i b_k) \log(z - z_k) + f_0(z), \tag{4.18}
$$

where a_k and b_k are real constants, and f_0 is single-valued and analytic on Σ. Thus, we obtain

$$
\begin{aligned}
&[w_1 + i w_2]_{L_k} = 2\pi i (z B_k + a_k + i b_k), \\
&[F]_{L_k} = 2\pi i B_k,
\end{aligned}
\tag{4.19}
$$

where $[g]_{L_k}$ denotes the change in value of the function g inside on passing once round a contour in the conventional positive sense which keeps the area enclosed on the left, the contour lying entirely in the body. Since the functions u_α, φ and ψ are single-valued, it follows that the functions u_α^0 and φ^0 are multi-valued

$$\begin{aligned} \left[u_1^0\right]_{L_k} &= 2\pi A_1(B_k x_2 + b_k), \\ \left[u_2^0\right]_{L_k} &= -2\pi A_1(B_k x_1 + a_k), \\ \left[\varphi^0\right]_{L_k} &= -2\pi A_1 B_k, \quad [\psi^0]_{L_k} = 0. \end{aligned} \tag{4.20}$$

In this way, the thermoelastic problem associated to the multiply-connected domain Σ reduces to a plane problem of mechanical dislocations defined by the field equations (4.10), (4.12) and (4.14) on Σ, the boundary conditions (4.15) on L and the dislocation characteristics

$$\varepsilon_k^0 = -2\pi A_1 B_k, \quad \alpha_k^0 = 2\pi A_1 b_k, \quad \beta_k^0 = -2\pi A_1 a_k. \tag{4.21}$$

Remark 2. Assume that
a) the domain Σ is multiply-connected;
b) the heat sources is absent;
c) the mechanical loads are absent.

Then the thermoelastic plane strain problem associated to Σ reduces to solving of the isothermal plane problem of dislocations corresponding to zero body loads, zero boundary traction, the boundary couple $\eta_1 dT/ds$, the microstretch traction $\eta_2 \partial T/\partial n$, and to the dislocation characteristics (4.21).

5. Thermoelastic deformation of a hollow cylinder

Let us study the thermoelastic problem for a tube. We assume that the domain Σ is defined by $\Sigma = \left\{\mathbf{x} : R_1^2 < x_1^2 + x_2^2 < R_2^2, x_3 = 0\right\}$ where R_1 and R_2 are positive constants. We suppose that the body is in equilibrium in the absence of mechanical loads and heat sources. Further, we suppose that the temperature on the outer surface is T_2 and that the temperature on the inner surface is T_1, where T_1 and T_2 are given constants.

We denote $r = (x_1^2 + x_2^2)^{1/2}$. The thermal boundary conditions are

$$T = T_1 \text{ for } r = R_1, \quad T = T_2 \text{ for } r = R_2. \tag{5.1}$$

The solution of the equation (3.5) which satisfies the boundary conditions (5.1) is given by

$$T = C_1 \ln r + C_2, \tag{5.2}$$

where

$$C_1(\ln R_2 - \ln R_1) = T_2 - T_1, \quad C_2(\ln R_2 - \ln R_1) = T_1 \ln R_2 - T_2 \ln R_1. \tag{5.3}$$

The function F defined by (4.1) is

$$F(z) = C_1 \log z + C_2. \tag{5.4}$$

It follows from (4.2) and (5.4) that

$$w_1 + iw_2 = z(C_1 \log z + C_2 - C_1). \tag{5.5}$$

The constants B_1, a_1 and b_1 which appear in (4.18) are given by

$$B_1 = C_1, \quad a_1 = 0, \quad b_1 = 0.$$

Thus, according to Remark 2, in this case the thermoelastic problem reduces to the problem of mechanical dislocations defined by the equations (4.10), (4.12) and (4.14) on Σ, the dislocation characteristics

$$\varepsilon_1^0 = -2\pi A_1 C_1, \quad \alpha_1^0 = 0, \quad \beta_1^0 = 0 \tag{5.6}$$

and the boundary conditions:

$$\begin{aligned} &t^0_{\beta\alpha} n_\beta = 0, \quad m^0_{\alpha 3} n_\alpha = 0 \quad \text{for } r = R_1 \text{ and } r = R_2, \\ &\pi^0_\alpha n_\alpha = -\eta_2 \frac{1}{R_1} C_1 \ \text{ for } r = R_1, \ \pi^0_\alpha n_\alpha = \eta_2 \frac{1}{R_2} C_1 \text{ for } r = R_2. \end{aligned} \tag{5.7}$$

We shall study this problem by using the polar coordinates (r, θ). We denote by u and v the physical components of the displacement vector in polar coordinates

$$u + iv = (u_1^0 + iu_2^0)e^{-i\theta}. \tag{5.8}$$

In polar coordinates, the geometrical equations (4.10) can be written as

$$\begin{aligned} &e_{rr} = \frac{\partial u}{\partial r}, \ e_{\theta\theta} = \frac{1}{r}(u + \frac{\partial v}{\partial \theta}), \ e_{r\theta} = \frac{\partial v}{\partial r} - \varphi^0, \\ &e_{\theta r} = \frac{1}{r}(\frac{\partial u}{\partial \theta} - v) + \varphi^0, \ \kappa_{rz} = \frac{\partial \varphi^0}{\partial r}, \ \kappa_{\theta z} = \frac{1}{r}\frac{\partial \varphi^0}{\partial \theta}. \end{aligned} \tag{5.9}$$

The constitutive equations (4.12) can be expressed in the form

$$
\begin{aligned}
&t_{rr} = (\lambda + 2\mu + \kappa)e_{rr} + \lambda e_{\theta\theta} + \lambda_0 \psi^0, \\
&t_{\theta\theta} = \lambda e_{rr} + (\lambda + 2\mu + \kappa)e_{\theta\theta} + \lambda_0 \psi^0, \\
&t_{r\theta} = (\mu + \kappa)e_{r\theta} + \mu e_{\theta r}, \;\; t_{\theta r} = (\mu + \kappa)e_{\theta r} + \mu e_{r\theta}, \\
&m_{rz} = \gamma \frac{\partial \varphi^0}{\partial r} + \frac{1}{r} b_0 \frac{\partial \psi^0}{\partial \theta}, \; m_{\theta z} = \frac{1}{r}\gamma \frac{\partial \varphi^0}{\partial \theta} - b_0 \frac{\partial \psi^0}{\partial r}, \\
&\pi_r = a_0 \frac{\partial \psi^0}{\partial r} - \frac{1}{r} b_0 \frac{\partial \varphi^0}{\partial \theta}, \; \pi_\theta = \frac{1}{r} a_0 \frac{\partial \psi^0}{\partial \theta} + b_0 \frac{\partial \varphi^0}{\partial r}, \\
&\sigma^0 = \lambda_0 (e_{rr} + e_{\theta\theta}) + \lambda_1 \psi^0.
\end{aligned}
\tag{5.10}
$$

The equations of equilibrium (4.14) become

$$
\begin{aligned}
&\frac{\partial t_{rr}}{\partial r} + \frac{1}{r}\frac{\partial t_{\theta r}}{\partial \theta} + \frac{1}{r}(t_{rr} - t_{\theta\theta}) = 0, \\
&\frac{\partial t_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial t_{\theta\theta}}{\partial \theta} + \frac{1}{r}(t_{r\theta} + t_{\theta r}) = 0, \\
&\frac{\partial m_{rz}}{\partial r} + \frac{1}{r}\frac{\partial m_{\theta z}}{\partial \theta} + \frac{1}{r} m_{rz} + t_{r\theta} - t_{\theta r} = 0, \\
&\frac{1}{r}\frac{\partial}{\partial r}(r\pi_r) + \frac{1}{r}\frac{\partial \pi_\theta}{\partial \theta} - \sigma^0 = 0.
\end{aligned}
\tag{5.11}
$$

The boundary conditions (5.7) reduce to

$$
\begin{aligned}
&t_{rr} = 0, t_{r\theta} = 0, \;\; m_{rz} = 0 \;\; \text{for } r = R_1 \text{ and } r = R_2, \\
&\pi_r = \frac{1}{R_1}\eta_2 C_1 \;\; \text{for } r = R_1, \; \pi_r = \frac{1}{R_2}\eta_2 C_1 \;\; \text{for } r = R_2.
\end{aligned}
\tag{5.12}
$$

We seek the solution $(u, v, \varphi^0, \psi^0)$ in the form

$$
\begin{aligned}
&u = \Lambda r \ln r + rH(r), \\
&v = \frac{1}{2\pi}\varepsilon_1^0 r\theta, \\
&\varphi^0 = \frac{1}{2\pi}\varepsilon_1^0 \theta, \;\; \psi^0 = G(r),
\end{aligned}
\tag{5.13}
$$

where

$$\Lambda = \frac{2\mu + \kappa}{4\pi(\lambda + 2\mu + \kappa)}\varepsilon_1^0, \tag{5.14}$$

and H and G are unknown functions of class C^2. We note that the functions u, v, φ^0 and ψ^0 from (5.13) correspond to the dislocation characteristics (5.6). It follows from (5.9), (5.10) and (5.13) that

$$\begin{aligned}
t_{rr} &= (\lambda + 2\mu + \kappa)(\Lambda \ln r + \Lambda + H + rH') + \lambda(\Lambda \ln r + H + \frac{\varepsilon_1^0}{2\pi}) + \lambda_0 G, \\
t_{\theta\theta} &= \lambda(\Lambda \ln r + \Lambda + H + rH') + (\lambda + 2\mu + \kappa)(\Lambda \ln r + H + \frac{\varepsilon_1^0}{2\pi}) + \lambda_0 G, \\
t_{r\theta} &= t_{\theta r} = 0, \\
m_{rz} &= 0,\ m_{\theta z} = \frac{1}{2\pi r}\gamma\varepsilon_1^0, \\
\pi_r &= a_0 G' - \frac{1}{2\pi r} b_0 \varepsilon_1^0,\ \pi_\theta = 0, \\
\sigma^0 &= \lambda_0(2\Lambda \ln r + \Lambda + 2H + rH' + \frac{1}{2\pi}\varepsilon_1^0) + \lambda_1 G,
\end{aligned} \tag{5.15}$$

where $f'(r) = df(r)/dr$. The equations of equilibrium (5.11) reduce to

$$\begin{aligned}
&\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(r^2 H)\right] = -\frac{\lambda_0}{\lambda + 2\mu + \kappa}G', \\
&\frac{1}{r}\frac{d}{dr}(rG') - \frac{\lambda_1}{a_0}G - \frac{\lambda_0}{a_0 r}(r^2 H)' = \frac{2\lambda_0}{a_0}\Lambda \ln r + \Gamma,
\end{aligned} \tag{5.16}$$

where

$$\Gamma = \frac{2\lambda + 6\mu + 3\kappa}{4\pi a_0(\lambda + 2\mu + \kappa)}\varepsilon_1^0 \lambda_0. \tag{5.17}$$

The first equation of the system (5.16) implies that

$$(r^2 H)' = -\frac{\lambda_0}{\lambda + 2\mu + \kappa} rG + D_1 r, \tag{5.18}$$

where D_1 is an arbitrary constant. Thus, the second equation from (5.16) becomes

$$G'' + \frac{1}{r}G' - p^2 G = \frac{2\lambda_0}{a_0}\Lambda \ln r + \Gamma + \frac{\lambda_0}{a_0}D_1, \tag{5.19}$$

where

$$p = \left[\frac{\lambda_1(\lambda + 2\mu + \kappa) - \lambda_0^2}{a_0(\lambda + 2\mu + \kappa)}\right]^{1/2}. \tag{5.20}$$

It follows from (2.6) that $p^2 > 0$. The general solution of the equation (5.19) is given by

$$G = G_0(r) - \frac{1}{p^2}\left(\Gamma + \frac{\lambda_0}{a_0}D_1 + \frac{2\lambda_0}{a_0}\Lambda \ln r\right), \tag{5.21}$$

where

$$G_0(r) = C_1^* I_0(pr) + C_2^* K_0(pr), \tag{5.22}$$

C_α^* are arbitrary constants, and I_n and K_n are modified Bessel functions of order n. We note that

$$\frac{d}{dr}(rG_0') = p^2 r G_0.$$

From (5.18) we obtain

$$\begin{aligned} H = &-\frac{\lambda_0}{p^2(\lambda+2\mu+\kappa)r}G_0'(r) + \frac{\lambda_0^2\Lambda}{2a_0p^2(\lambda+2\mu+\kappa)}(2\ln r - 1)+ \\ &+\frac{1}{r^2}D_2 + \frac{1}{2}MD_1 + \frac{\lambda_0\Gamma}{2p^2(\lambda+2\mu+\kappa)}, \end{aligned} \tag{5.23}$$

where D_2 is an arbitrary constant and

$$M = \frac{\lambda_0^2}{p^2 a_0(\lambda+2\mu+\kappa)} + 1.$$

We note that

$$rH' = \frac{\lambda_0}{p^2(\lambda+2\mu+\kappa)}\left(\frac{2}{r}G_0' - p^2 G_0\right) + \frac{\lambda_0^2\Lambda}{a_0p^2(\lambda+2\mu+\kappa)} - \frac{2}{r^2}D_2. \tag{5.24}$$

It follows from (5.15), (5.21)-(5.24) that

$$\begin{aligned} t_{rr} = &\frac{(2\mu+\kappa)\lambda_0}{p^2(\lambda+2\mu+\kappa)r}G_0'(r) + \frac{d}{2p^2a_0}D_1 - \frac{2\mu+\kappa}{r^2}D_2+ \\ &+\frac{d}{p^2a_0}\Lambda\ln r + \frac{d}{4\pi a_0 p^2}\varepsilon_1^0, \\ \pi_r = &\, a_0p[C_1^*I_1(pr) - C_2^*K_1(pr)] - \frac{2\lambda_0\Lambda}{p^2 r} + \frac{b_0}{2\pi r^2}\varepsilon_1^0, \end{aligned} \tag{5.25}$$

where d is defined in (4.5). The boundary conditions for π_r reduces to

$$\begin{aligned} C_1^* &= [g(R_2)K_1(pR_1) - g(R_1)K_1(pR_2)]\,N, \\ C_2^* &= [g(R_2)I_1(pR_1) - g(R_1)I_1(pR_2)]\,N, \end{aligned} \tag{5.26}$$

where

$$g(r) = \frac{1}{a_0 p}\left(\frac{2\lambda_0}{p^2 r}\Lambda - \frac{b_0}{2\pi r^2}\varepsilon_1^0 + \frac{1}{r}\eta_2 C_1\right),$$

$$N^{-1} = I_1(pR_2)K_1(pR_1) - I_1(pR_1)K_1(pR_2).$$

The remaining boundary conditions are satisfied if the constants D_1 and D_2 have values

$$\begin{aligned} D_1 &= (2\mu+\kappa)\tilde{H}\left[\frac{1}{R_1^2}h(R_2) - \frac{1}{R_2^2}h(R_1)\right], \\ D_2 &= \frac{d\tilde{H}}{2p^2 a_0}[h(R_2) - h(R_1)], \end{aligned} \tag{5.27}$$

where

$$\tilde{H}^{-1} = \frac{(2\mu+\kappa)d}{2p^2 a_0 R_1^2 R_2^2}(R_2^2 - R_1^2),$$

$$h(r) = -\frac{d}{4\pi a_0 p^2}\varepsilon_1^0 - \frac{d}{p^2 a_0}\Lambda \ln r - \frac{(2\mu+\kappa)\lambda_0}{p^2(\lambda+2\mu+\kappa)r}G_0'(r).$$

In this way the functions H and G are completely determined. Thus, the solution of the isothermal plane strain problem is given by (5.13) and the solution of the thermoelastic problem follows from (4.9), (5.5), (5.8) and (5.13).

6. Thermal stresses in a cylinder subjected to a uniform temperature gradient

Let B be a right cylinder with the cross-section Σ defined by $\Sigma = \{\mathbf{x} : x_1^2 + x_2^2 < a^2, x_3 = 0\}$ where a is a positive constant. The cylinder is in equilibrium in the absence of mechanical loads and heat sources. We suppose that the heat flux $\widetilde{q}$ associated to the boundary of Σ is kQn_1, where Q is a given constant. In this case the temperature field T is the solution of the equation (3.5) on Σ with the boundary condition

$$\frac{\partial T}{\partial n} = Qn_1 \quad \text{for} \quad r = a. \tag{6.1}$$

The solution of this boundary value problem is given by

$$T = Qr\cos\theta, \tag{6.2}$$

where r and θ are the polar coordinates. The function F defined by (4.1) has the form

$$F(z) = Qz. \tag{6.3}$$

It follows from (4.2) and (6.3) that

$$w_1 = \frac{1}{2}Qr^2 \cos 2\theta, \quad w_2 = \frac{1}{2}Qr^2 \sin 2\theta. \tag{6.4}$$

On the basis of Remark 1 the thermoelastic problem reduces to the solving of the isothermal elastic plane strain problem defined by the equations (4.10), (4.12) and (4.14) on Σ, and the boundary conditions

$$\begin{aligned} &t^0_{\beta\alpha}n_\beta = 0, \quad m^0_{\alpha 3}n_\alpha = -\eta_1 Q n_2, \\ &\pi^0_\alpha n_\alpha = \eta_2 Q n_1 \quad \text{for} \quad r = a. \end{aligned} \tag{6.5}$$

The boundary conditions (6.5) can be expressed with the help of notations (5.10) in the form

$$\begin{aligned} &t_{rr} = 0, \quad t_{r\theta} = 0, \quad m_{rz} = -\eta_1 Q \sin\theta, \\ &\pi_r = \eta_2 Q \cos\theta \quad \text{for} \quad r = a. \end{aligned} \tag{6.6}$$

We seek the solution of the isothermal problem in the form

$$\begin{aligned} &u = U(r)\cos\theta, \quad v = V(r)\sin\theta, \\ &\varphi^0 = W(r)\sin\theta, \psi^0 = \Psi(r)\cos\theta, \end{aligned} \tag{6.7}$$

where U, V, W and Ψ are functions only on r. From (5.9), (5.10) and (6.7) we obtain

$$\begin{aligned} t_{rr} &= [(\lambda + 2\mu + \kappa)U' + \frac{1}{r}\lambda(U + V) + \lambda_0\Psi]\cos\theta, \\ t_{\theta\theta} &= [\lambda U' + (\lambda + 2\mu + \kappa)\frac{1}{r}(U + V) + \lambda_0\Psi]\cos\theta, \\ t_{r\theta} &= [(\mu + \kappa)V' - \frac{1}{r}\mu(U + V) - \kappa W]\sin\theta, \\ t_{\theta r} &= [\mu V' - \frac{1}{r}(\mu + \kappa)(U + V) + \kappa W]\sin\theta, \\ m_{rz} &= \left(\gamma W' - \frac{1}{r}b_0\Psi\right)\sin\theta, \\ m_{\theta z} &= \left(\frac{1}{r}\gamma W - b_0\Psi'\right)\cos\theta, \\ \pi_r &= \left(a_0\Psi' - \frac{1}{r}b_0 W\right)\cos\theta, \quad \pi_\theta = \left(b_0 W' - \frac{1}{r}a_0\Psi\right)\sin\theta, \\ \sigma^0 &= \left\{\lambda_0[U' + \frac{1}{r}(U + V)] + \lambda_1\Psi\right\}\cos\theta. \end{aligned} \tag{6.8}$$

It follows from (5.11) and (6.8) that the functions U, V, W and Ψ satisfy the following system of equations

$$
\begin{aligned}
& r^2\frac{d^2U}{dr^2}+r\frac{dU}{dr}-(1+c_1)U+(1-c_1)r\frac{dV}{dr}- \\
& \qquad\qquad -(1+c_1)V+c_3rW-c_2r^2\frac{d\Psi}{dr}=0, \\
& c_1\left(r^2\frac{d^2V}{dr^2}+r\frac{dV}{dr}\right)-(1+c_1)V-(1-c_1)r\frac{dU}{dr}- \\
& \qquad\qquad -(1+c_1)U-c_3r^2\frac{dW}{dr}-c_2r\Psi=0, \\
& r^2\frac{d^2W}{dr^2}+r\frac{dW}{dr}-(1+2c_4r^2)W+c_4r^2\frac{dV}{dr}+c_4rV+c_4rU=0, \\
& a_0\left[\frac{1}{r}\frac{d}{dr}(r\frac{d\Psi}{dr})-\frac{1}{r^2}\Psi\right]-\lambda_0\left[\frac{1}{r}\frac{d}{dr}(rU)+\frac{1}{r}V\right]-\lambda_1\Psi=0,
\end{aligned}
\tag{6.9}
$$

where

$$
c_1=\frac{\mu+\kappa}{\lambda+2\mu+\kappa},\quad c_2=\frac{\lambda_0}{\lambda+2\mu+\kappa},\quad c_3=\frac{\kappa}{\lambda+2\mu+\kappa},\quad c_4=\frac{\kappa}{\gamma}. \tag{6.10}
$$

Let us introduce the independent variable t through the relation

$$
t=\ln r. \tag{6.11}
$$

If we denote $D=d/dt$ then the first two equations (6.9) can be written in the form

$$
\begin{aligned}
& [D^2-(1+c_1)]U+[(1-c_1)D-(1+c_1)]V=-e^t(c_3W+c_2D\Psi), \\
& \left[(c_1-1)D-(1+c_1)\right]U+\left[c_1D^2-(1+c_1)\right]V=e^t(c_3DW+c_2\Psi).
\end{aligned}
\tag{6.12}
$$

The general solution of the homogeneous system (6.12) which correspond to a non-rigid displacement is given by

$$
\begin{aligned}
& U_0=E_1t+E_2e^{2t}+E_3e^{-2t}, \\
& V_0=-\frac{1-c_1}{1+c_1}E_1-E_1t-\frac{3-c_1}{1-3c_1}E_2e^{2t}+E_3e^{-2t},
\end{aligned}
\tag{6.13}
$$

where E_j are arbitrary constants. A particular solution of nonhomogeneous system is

$$U^* = -\frac{c_3}{2c_1}(S_1 - S_2 e^{-2t}) - \frac{1}{2}c_2(S_3 + S_4 e^{-2t}),$$
$$V^* = \frac{c_3}{2c_1}(S_1 + S_2 e^{-2t}) + \frac{1}{2}c_2(S_3 - S_4 e^{-2t}), \tag{6.14}$$

where

$$S_1(t) = \int^t e^x W(x)dx, \quad S_2(t) = \int^t e^{3x} W(x)dx,$$
$$S_3(t) = \int^t e^x \Psi(x)dx, \quad S_4(t) = \int^t e^{3x}\Psi(x)dx. \tag{6.15}$$

In view of (6.13) - (6.15) we find that

$$U = E_1 \ln r + E_2 r^2 + E_3 r^{-2} - \frac{c_3}{2c_1}\left(\int^r W(x)dx - r^{-2}\int^r x^2 W(x)dx\right) -$$
$$-\frac{1}{2}c_2\left(\int^r \Psi(x)dx + r^{-2}\int^r x^2\Psi(x)dx\right),$$
$$V = -\frac{1-c_1}{1+c_1}E_1 - E_1\ln r - \frac{3-c_1}{1-3c_1}E_2 r^2 + E_3 r^{-2} +$$
$$+\frac{c_3}{2c_1}\left(\int^r W(x)dx + r^{-2}\int^r x^2 W(x)dx\right) +$$
$$+\frac{1}{2}c_2\left(\int^r \Psi(x)dx - r^{-2}\int^r x^2\Psi(x)dx\right). \tag{6.16}$$

Since U and V must be finite for $r = 0$ we conclude that $E_1 = 0$ and $E_3 = 0$. If we substitute U and V from (6.16) into the third equation of (6.9) then we obtain

$$r^2\frac{d^2W}{dr^2} + r\frac{dW}{dr} - (1+\zeta^2 r^2)W = \frac{8}{1-3c_1}c_4 E_2 r^3, \tag{6.17}$$

where

$$\zeta = \left[\frac{\kappa(2\mu+\kappa)}{\gamma(\mu+\kappa)}\right]^{1/2}. \tag{6.18}$$

The solution of this equation is

$$W = E_4 K_1(\zeta r) + E_5 I_1(\zeta r) - \frac{8(\mu+\kappa)}{(2\mu+\kappa)(1-3c_1)}E_2 r, \tag{6.19}$$

where E_4 and E_5 are arbitrary constants. Since W must be finite for $r = 0$ we have $E_4 = 0$. If we substitute U and V from (6.16) into the last equation from (6.9) we find the following equation for Ψ

$$r^2 \frac{d^2\Psi}{dr^2} + r\frac{d\Psi}{dr} - (1 + p^2r^2)\Psi = -\frac{8}{a_0(1-3c_1)} c_1\lambda_0 E_2 r^3,$$

where p has been defined in (5.20). The solution which is finite for $r = 0$ is given by

$$\Psi = E_6 I_1(pr) + \frac{8}{a_0 p^2(1-3c_1)} c_1\lambda_0 E_2 r, \tag{6.20}$$

where E_6 is an arbitrary constant. It follows from (6.16), (6.19) and (6.20) that the solution of the system (6.9) can be expressed in the form

$$\begin{aligned} U = {} & \Lambda_1 E_2 r^2 - \frac{1}{2c_1\zeta} c_3 E_5 [I_0(\zeta r) - I_2(\zeta r)] - \\ & - \frac{1}{2p} c_2 E_6 [I_0(pr) + I_2(pr)], \\ V = {} & -\Lambda_2 E_2 r^2 + \frac{1}{2c_1\zeta} c_3 E_5 [I_0(\zeta r) + I_2(\zeta r)] + \\ & + \frac{1}{2p} c_2 E_6 [I_0(pr) - I_2(pr)], \end{aligned} \tag{6.21}$$

$$W = E_5 I_1(\zeta r) - \Lambda_3 E_2 r, \quad \Psi = E_6 I_1(pr) + \Lambda_4 E_2 r,$$

where

$$\begin{aligned} \Lambda_1 &= \frac{1}{1-3c_1}\left(1 - 3c_1 + \frac{\kappa}{2\mu+\kappa} - \frac{c_1c_2\lambda_0}{a_0p^2}\right), \\ \Lambda_2 &= \frac{1}{1-3c_1}\left(3 - c_1 + \frac{3\kappa}{2\mu+\kappa} - \frac{c_1c_2\lambda_0}{a_0p^2}\right), \\ \Lambda_3 &= \frac{8(\mu+\kappa)}{(2\mu+\kappa)(1-3c_1)}, \quad \Lambda_4 = \frac{8c_1\lambda_0}{a_0(1-3c_1)p^2}. \end{aligned} \tag{6.22}$$

In view of (6.8) and (6.21) we obtain

$$t_{rr} = \left[\Gamma_1 E_2 r - \frac{1}{r}\zeta\gamma E_5 I_2(\zeta r) + \frac{2\mu+\kappa}{pr} c_2 E_6 I_2(pr)\right]\cos\theta,$$

$$t_{r\theta} = \left[-\Gamma_2 E_2 r - \frac{1}{r}\zeta\gamma E_5 I_2(\zeta r) + \frac{2\mu+\kappa}{pr} c_2 E_6 I_2(pr)\right]\sin\theta,$$

$$m_{rz} = \left\{\gamma\zeta E_5 I_1'(\zeta r) - \frac{1}{r}E_6 b_0 I_1(pr) - P_1 E_2\right\}\sin\theta, \tag{6.23}$$

$$\pi_r = \left\{a_0 p E_6 I_1'(pr) - \frac{1}{r}E_5 b_0 I_1(\zeta r) + P_2 E_2\right\}\cos\theta,$$

where

$$\begin{aligned}\Gamma_1 &= (3\lambda+4\mu+2\kappa)\Lambda_1 - \lambda\Lambda_2 + \lambda_0\Lambda_4,\\ \Gamma_2 &= \mu\Lambda_1 + (\mu+2\kappa)\Lambda_2 - \kappa\Lambda_3,\\ P_1 &= \gamma\Lambda_3 + b_0\Lambda_4, \quad P_2 = b_0\Lambda_3 + a_0\Lambda_4.\end{aligned} \tag{6.24}$$

We can see that $\Gamma_1 = -\Gamma_2$ so that the first two boundary conditions (6.6) imply that

$$E_2 = \frac{1}{a^2\Gamma_1}\zeta\gamma E_5 I_2(\zeta a) - \frac{2\mu+\kappa}{pa^2\Gamma_1}c_2 E_6 I_2(pa). \tag{6.25}$$

The remaining boundary conditions from (6.6) determine the constants E_5 and E_6

$$E_5 = -\frac{Q}{P_3}\Gamma_1(R_{12}\eta_2 + R_{22}\eta_1), \quad E_6 = \frac{Q}{P_3}\Gamma_1(R_{11}\eta_2 + R_{21}\eta_1), \tag{6.26}$$

where

$$R_{11} = \frac{1}{2}\gamma\zeta\Gamma_1 I_0(\zeta a) + \gamma\zeta\left(\frac{1}{2}\Gamma_1 - \frac{1}{a^2}P_1\right)I_2(\zeta a),$$

$$R_{12} = \frac{2\mu+\kappa}{pa^2}P_1 c_2 I_2(pa) - \frac{1}{a}\Gamma_1 b_0 I_1(pa),$$

$$R_{21} = \frac{1}{a^2}\gamma\zeta P_2 I_2(\zeta a) - \frac{1}{a}\Gamma_1 b_0 I_1(pa),$$

$$R_{22} = \frac{1}{2}a_0 p\Gamma_1 I_0(pa) + \left[\frac{1}{2}a_0 p\Gamma_1 - \frac{2\mu+\kappa}{pa^2}P_2 c_2\right]I_2(pa),$$

$$P_3 = R_{11}R_{22} - R_{12}R_{21}.$$

Thus, the solution of the isothermal plane strain problem is given by (6.7) and (6.21), where the constants E_2, E_5 and E_6 can be determined from (6.25) and (6.26). By (5.8) and (6.7), we get

$$u_1^0 = U\cos^2\theta - V\sin^2\theta, \quad u_2^0 = \frac{1}{2}(U+V)\sin 2\theta.$$

In view of (6.4), (4.9) and (6.3) we obtain the solution of the thermoelastic problem in the form

$$u_1 = (U + \frac{1}{2}A_1Qr^2)\cos^2\theta - (V + \frac{1}{2}A_1Qr^2)\sin^2\theta,$$

$$u_2 = \frac{1}{2}(U + V + A_1Qr^2)\sin 2\theta,$$

$$\varphi = (W + A_1Qr)\sin\theta, \quad \psi = (\Psi + A_2Qr)\cos\theta.$$

We note that in the classical theory of thermoelasticity [7] the plane stresses generated by the temperature field (6.2) are equal to zero.

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