Integral domains in which each ideal is a w-ideal

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Abstract

In this paper, we investigate integral domains in which each ideal is a w-ideal (i.e. the d- and w-operations are the same), called the DW-domains. In some sense this study is similar to that one given in [9] for the TV-domains. We prove that a domain R is a DW-domain if and only if each maximal ideal of R is a w-ideal, and if R is a domain such that R_M is a DW-ideal for each maximal ideal M of R, then so is R, and the equivalence holds when R is v-coherent. We describe the w-operation on pullbacks in order to provide original examples. MSC:13G05, 13A15, 13F05.

Introduction

The notion of the w-operation has recently received much more interest for its homological aspect. This paper aims to investigate the ideal-theoretic properties of this star-operation. It is well known that for a domain R, $d \leq w \leq t \leq v$ in the sense that for each nonzero fractional ideal I of R, $I = I_d \subseteq I_w \subseteq I_t \subseteq I_v$, and the inclusions may be strict ([12, Proposition 2.3). In [8], W. Heinzer has initiated the study of domains in which each ideal is divisorial (i.e. each ideal is a v-ideal, or $d = v$) and called them divisorial domains. Inspired from this work, E. Houston and M. Zafrullah studied the so-called TV domains, i.e. domains in which each t-ideal is a v-ideal (or, $t = v$). Recently, we have studied the TW domains, i.e. domains

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in which each w-ideal is a t-ideal, or $w = t$. Our investigation follows this way in studying what we so-called DW-domains, or domains in which each ideal is a w-ideal, i.e. the d- and w-operations are the same. In the second section, we motivate our study by constructing examples showing that the notions of DW-domains, TW-domains and TV-domains are completely independent. Following [9], the equality of the t and v-operations on maximal ideals of a domain R is not sufficient for R to be a TV-domain. However, we prove that a domain R is a DW-domain if and only if if each maximal ideal is a w-ideal (i.e the equality of the d- and w-operations on maximal ideals forces the equality of the two star-operations) (Proposition 2.2). This will be the first step in remarking the difference between these two classes of domains. We Also prove that if R_M is a DW-domain for each maximal ideal M of R, then so is R, and the equivalence holds when R is v-coherent. The section close with a complete characterization of Noetherian domain for which each overring is a DW-domain. It turns out that such domain is of one dimensional. The third section deals with pullbacks in order to provide original examples. We start by examining the situation for the diagram of type $(\sqcap)$ in which M is assumed to be a maximal ideal of T. We then examine the larger case of diagram of type (Δ), where M is not necessary maximal in T. We prove that for such diagram, assume that $qf(D) \subseteq T/M$. If T and D are DW-domains, then so is R (Theorem 3.6). The section close with a description of the composite ring $R = A + XB[[X]]$, which is a particular case of diagram of type (Δ), but in which the assumption $T = B[[X]]$ is a DW-domain is not required since this forces B to be a field, Proposition 2.6.

2. Motivation and Preliminaries.

Example 2.1

1) A DW-domain which is neither a TW-domain nor a TV-domain.

Let Q denote the rational number field, Z the ring of integers and X and Y indeterminates over Q. Set $V = Q(X)[[Y]] = Q(X) + M$ where $M = YV$ and set $R = Z + M$. Then each maximal ideal N of R is of the form $N = pZ + M = pR$, where p is prime (i.e. irreducible) in Z. So N is a principal ideal of R and therefore a v-ideal. Since $N^{-1} = p^{-1}R \supset R$, then R is a DW-domain, [12, Proposition 1.3]). Since R is an integrally closed domain which is not a $PVMD$, then R is not a TW-domain, and since M is a v-ideal of R which is contained in infinitely many t-maximal ideals (clearly each maximal ideal N of R contains M), then R is not a TV-domain ([9, Theorem 1.3]).

2) A TW-domain which is neither a DW-domain nor a TV-domain.

Let A be a $PVMD$ which is not a field with at least one non t-invertible t-maximal ideal M and let X be an indeterminate over A. Set $R = A[X]$. Then R is a $PVMD$ and therefore a TW-domain. Since A is not a field, by Proposition 2.6 below, R is not a DW-domain. Now, since M is not t-invertible in A, then $MM^{-1} = M$. So $M^{-1} = (M : M) = A$ and then $M_v = A$. Since $(M[X])_t = M_t[X] = M[X]$ and $(M[X])_v = M_v[X] = A[X] = R$, then $M[X]$ is a t-ideal of R which is not a v-ideal. It follows that R is not a TV-domain.

3) A TV-domain which is neither a DW-domain nor a TW-domain.

Let A be an integrally closed domain which is not a $PVMD$ and for which $A[X]$ is a Mori domain. Set $R = A[X]$. Since R is Mori, then R is a TV-domain. Since A is not a field, by Proposition 2.6 below, R is not a DW-domain. Finally since R is an integrally closed domain which is not a $PVMD$, then R is not a TW-domain.

Proposition 2.2. Let R be an integral domain. The following conditions are equivalent.

i) R is a DW-domain;
ii) Every prime ideal of R is a w-ideal;
iii) Every maximal ideal of R is a w-ideal;
iv) Every maximal ideal of R is a t-ideal;
v) $GV(R) = \{R\}$.

Proof. *i*) $\Longrightarrow$ *ii*) $\Longrightarrow$ *iii*) Trivials.
iii) $\Longrightarrow$ *iv*) Follows from [11, Lemma 2.1]
iv) $\Longrightarrow$ *v*). Let $A \in GV(R)$. If $A \subset R$, then let M be a maximal ideal of R which contains A. So $R = A_w = A_t \subseteq M_t = M$, by *iv*) which is absurd.
v) $\Longrightarrow$ *i*) Let I be a nonzero ideal of R and let $x \in I_w$. Then there exists $A \in GV(R)$ such that $xA \subseteq I$. By *v*), $A = R$. Hence $xR \subseteq I$ and therefore $x \in I$. It follows that $I_w = I$ and therefore R is a DW-domain.

Corollary 2.3. Let R be a domain with $t - \dim R = 1$. Then R is a DW-domain if and only if $\dim R = 1$. In particular a Krull domain is a DW-domain if and only if it is a Dedekind domain.

Proof. Let $X^1(R)$ denote the set of prime ideals of R of height one. Since $t - \dim R = 1$, then $X^1(R) = t - Max(R)$. Since R is a DW-domain, by Proposition 2.2, $Max(R) = t - Max(R)$ Hence $X^1(R) = Max(R)$ and therefore $\dim R = 1$.

Now if R is a Krull domain which is a DW-domain, then $\dim R = 1$ and therefore R is a Dedekind domain.

According to [12, Proposition 1.3], if for each maximal ideal M of R, $M^-1 \supset R$, then R is a DW-domain. The converse is not true. Indeed, let R be an almost Dedekind domain which is not Dedekind. Since R is a Prüfer domain, then R is a DW-domain (in fact $d = w = t$). Since $\dim R = 1$ and R is not Dedekind, then R has a non invertible maximal ideal M. Hence $MM^{-1} = M$. So $M^{-1} = (M : M) = R$, as desired. The following corollary states when the equivalence holds.

Corollary 2.4. Let R be a H-domain. The following conditions are equivalent.

i) R is a DW-domain;

ii) $M^{-1} \supset R$ for each maximal ideal M of R (i.e each maximal ideal of R is divisorial);

iii) $I^{-1} \supset R$ for each proper ideal I of R.

Proof. *i)* $\Longrightarrow$ *ii)* Suppose that $M^{-1} = R$ for some maximal ideal M of R.Since R is a H-domain, then there is a f.g. ideal $I \subseteq M$ such that $I^{-1} = M^{-1} = R$. Hence $I \in GV(R)$ and therefore $I = R$ by Proposition 2.2, which is absurd.

ii) $\Longrightarrow$ *iii)* Let I be a proper ideal of R and let M be a maximal ideal of R with $I \subseteq M$. Then clearly $R \subset M^{-1} \subseteq I^{-1}$.

iii) $\Longrightarrow$ *i)* Follows from Proposition 2.2 since $I^{-1} \supset R$ for each proper ideal I of R forces that R is the unique ideal J of such that $J^{-1} = R$. Hence $GV(R) = \{R\}$, and then R is a DW-domain.

Corollary 2.5. Let R be a semikrull domain. Then R is a DW domain if and only if $\dim R = 1$.

Proposition 2.6. Let R be an integral domain and let X be an indeterminate over R. Then $R[X]$ (respectively $R[[X]]$) is a DW-domain if and only if R is a field.

Proof. Assume that $R[X]$ is a DW-domain. Let $0 \neq d \in R$ and let J be the finitely generated ideal of R given by $J = dR + XR$. We claim that $J^1 = (R[X] : J) = R[X]$. Indeed, Since $dR[X] \subseteq J$, then $J^{-1} \subseteq d^{-1}R[X]$. Let $f \in J^{-1}$ and write $f = d^{-1}g$ for some $g = \sum_{i=0}^{i=n} a_iX^i \in R[X]$. Since $X \in J$, then $d^{-1}gX = fX \in R[X]$. Hence, for each $i \in \{1, \ldots, n\}$, $d^{-1}a_i \in R$. So $a_i \in dR$ and therefore $g \in dR[X]$. Hence $f = d^{-1}g \in R[X]$ and therefore $J^{-1} = R[X]$. Then $J_t = J_v = R[X]$. Hence $J = J_w = R[X]$. So $d^{-1} \in R$ and therefore R is a field.

For $R[[X]]$, the proof is similar to $R[X]$.

Proposition 2.7. Let R be an integral domain. If R_M is a DW-domain for each maximal ideal M of R, then R is a DW-domain. Moreover, if R is v-coherent, then the equivalence holds.

Proof. Let I be a nonzero ideal of R and let $x \in I_w$. Then there is $A \in GV(R)$ such that $xA \subseteq I$. Now, for each maximal ideal M of R, since $AR_M \in GV(R_M)$ and $xAR_M \subseteq IR_M$, then $x \in (IR_M)_{w_1} = IR_M$ since R_M is a DW-domain. Hence $x \in \cup_{M \in Max(R)} IR_M = I$ and therefore $I = I_w$. It follows that R is a DW-domain.

Now, assume that R is v-coherent and R is a DW-domain. Let M be a maximal ideal of R. By Proposition 2.2, it suffices to show that MR_M is a w-ideal of R_M. Let $x \in (MR_M)_{w_1}$. Then there is $J \in GV(R_M)$ such that $xJ \subseteq MR_M$. Set $J = AR_M$ for some finitely generated ideal A of R. Since $xAR_M = xJ \subseteq MR_M$ and A is f;g, then $x\mu A \subseteq M$ for some $\mu \in R \setminus M$. So $x\mu A_v = x\mu A_t \subseteq M_t = M$, by Proposition 2.2. Hence $xA_vR_M = x\mu A_vR_M \subseteq MR_M$. Since R is v-coherent and A is f.g, then $A_vR_M = (AR_M)_{v_1} = J_{v_1} = R_M$. Hence $xR_M \subseteq MR_M$ and therefore $x \in MR_M$. It follows that MR_M is a w-ideal of R_M and therefore R_M is a DW-domain.

Example 2.8. A localization of a DW-domain is not necessarily a DW-domain. Indeed, let Q denotes the field of rational numbers and let X, Y and Z indeterminates over Q. Set $T = Q(\sqrt{2})[[X, Y, Z]] = Q(\sqrt{2}) + M$ and $R = Q + M$. By [6, Theorem 4.12], R is a local Noetherian domain. Let $P_1 = XT$, $P_2 = (X, Y)T$ and consider the chain $(0) \subset P_1 \subset P_2 \subset M$. Since $ht_T P_2 = 2$ and T is a Krull domain, then $T = (T : P_2) = (P_2 : P_2) = P_2^{-1}$. Hence $P_2^{-1} = T = M^{-1}$ and then $(P_2)_t = (P_2)_v = M$. So P_2 is not a t-ideal of R. Now, it is easy to see that $R_{P_2} = T_{P_2}$ is not a DW-domain since it is a Krull (so $PVMD$) which is not Prüfer. However since R is local with maximal ideal M and $R \subset T = M^{-1}$, then R is a DW-domain.

Theorem 2.9. Let R be a Noetherian domain. The following statements are equivalent.

i) Each overring of R is a DW-domain.

ii) $\dim R = 1$.

Proof. Since R is Noetherian, then $R' = \bar{R}$ is a Krull domain. By [11, Corollary 2.5], $w = t = v$. Since $R' = \bar{R}$ is a DW-domain, then $d = w$. Therefore R' is divisorial. By [8, Theorem 2.5], R is a Prüfer domain and therefore a Dedekind domain. Hence $\dim R = \dim R' = 1$, as desired.

Conversely, assume that $\dim R = 1$. Let T be an overring of R with $T \subset Qf(R)$. By [10, Theorem 93], T is Noetherian and $\dim T = 1$. Let I be a nonzero ideal of T and let $x \in I_{w_1}$. Then there exists $A \in GV(T)$

such that $xA \subseteq I$. If $A \subset T$, since $\dim T = 1$, then $GradA = 1$. So $A^{-1} = (T : A) \supset T$ ([10, Exercice2, page 102]), which is absurd since $A \in GV(T)$. Hence $A = T$ and therefore $x \in I$. It follows that $I = I_{w_1}$ and T is a DW-domain.

3. Pullbacks.

In this section we examine the transfer of the "DW" property to the pullbacks. To avoid unnecessary repetition, let us fix notation for the rest of this section. Data will consist of a pullback of canonical homomorphisms

$$\begin{array}{ccc} R & \longrightarrow & D \\ \downarrow & & \downarrow \\ T & \longrightarrow & T/M \end{array}$$

where T is an integral domain, M is an ideal of T (not necessarily maximal), $\varphi\colon T \to T/M$ is the natural projection, D is a proper domain contained in T/M, and $R = \varphi^{-1}(D)$. We explicitly assume that $R \subset T$ and we shall refer to this as a diagram of type (Δ). If M is a maximal ideal of T, we shall refer to this as a diagram of type $(\sqcap)$. The case where $T = V$ is a valuation domain of the form $K + M$, where K is a field and M is the maximal ideal of V is of particular interest. We shall refer to this as the classical $D + M$ construction.

Theorem 3.1. For the diagram of type $(\sqcap)$:

1) If R is a DW-domain, then so is D.
2) If T is local, then R is a DW-domain if and only if so is D.
3) If T and D are DW-domains, then so is R.

Proof. We need the following two lemmas.

Lemma 3.2 (cf. [4, Proposition 3.1]). For the diagram of type (Δ), assume that $qf(D) \subseteq T/M$. Then T is t-linked over R.

Proof. Let A be a nonzero ideal of R such that $A^{-1} = R$. Let $x \in (T : AT)$. Then $xAT \subseteq T$ implies that $xAM \subseteq M$. So $xM \subseteq (M : A) \subseteq (R : A) = R$. Since $A^{-1} = R \subset T \subseteq (M : M) \subseteq M^{-1}$, then $A \not\subseteq M$. Let $R_0 = \varphi^{-1}(k)$, where $k = qf(D)$, and consider the following diagram:

$$\begin{array}{ccc} R & \longrightarrow & D \\ \downarrow & & \downarrow \\ R_0 & \longrightarrow & k \\ \downarrow & & \downarrow \\ T & \longrightarrow & T/M \end{array}$$

If $AR_0 = R_0$, then $1 = \sum_{i=1}^{i=n} a_i x_i$, where $a_i \in A$ and $x_i \in R_0$ for each $i \in \{1, \dots, n\}$. Then $x = \sum_{i=1}^{i=n} x a_i x_i$. Since $xa_i \in xA \subseteq xAT \subseteq T$ and $x_i \in R_0 \subseteq T$ for each $i \in \{1, \dots, n\}$, then $xa_i x_i \in T$ and therefore $x \in T$. Assume that $AR_0 \subset R_0$. Since M is a maximal ideal of R_0 and $A \not\subseteq M$, then $AR_0 + M = R_0$. Hence $1 = a + m$ for some $a \in R_0$ and $m \in M$. So $x = xa + xm$. Since $xa \in xAR_0 \subseteq xAT \subseteq T$ and $xm \in xM \subseteq R \subseteq T$, then $x \in T$. It follows that $(T : AT) = T$ and therefore T is t-linked over R.

Lemma 3.3 ([12, Lemma 3.1]). For the diagram of type ($\sqcap$), let J be a (fractional) ideal of D and let $I = \varphi^{-1}(J)$. Then $I_w = \varphi^{-1}(J_w)$.

Proof of the Theorem. By Lemma 3.3, it is clear that if R is a DW-domain then so is D. So, we may assume that D is a DW-domain and prove that R is a DW-domain in the two cases. By Proposition 2.2, it suffices to show that each maximal ideal Q of R is a w-ideal. Let Q be a maximal ideal of R.

2) Assume that T is local with maximal ideal M. Then it is well known that each (integral) ideal of R is comparable (under inclusion) to M. By maximality, $M \subset Q$. Then $Q = \varphi^{-1}(q)$ for some maximal ideal q of D. Since D is a DW-domain, then $q = q_w$. By Lemma 3.3, $Q_w = \varphi^{-1}(q_w) = \varphi^{-1}(q) = Q$. Hence R is a DW-domain.

3) Assume that T is a DW-domain. If $M \subset Q$, then $Q = \varphi^{-1}(q)$ for some maximal ideal q of D. Since D is a DW-domain, by Lemma 3.3, $Q_w = \varphi^{-1}(q_w) = \varphi^{-1}(q) = Q$. Assume that $M \not\subseteq Q$. Then $Q + M = R$. So $1 = a + m$ for some $a \in Q$ and $m \in M$. Now, for each $x \in Q_w$, $x = xa + xm$. Since $xa \in Q$, to show that $x \in Q$ it suffices to show that $xm \in Q$. Since $x \in Q_w$, then there is $A \in GV(R)$ such that $xA \subset Q$. So $xAT \subseteq QT$. Since T is t-linked over R (Lemma 3.2), then $AT \in GV(T)$. Hence $x \in (QT)_{w_1} = QT$, since T is a DW-domain. So $xm \in QTM = QM \subseteq Q$, as desired. It follows that R is a DW-domain.

Example 3.4. 1) In the case where T is local, R is a DW-domain do not forces that T is a DW-domain. Indeed, let Q be the field of rational numbers and let X and Y indeterminates over Q. $T = Q(\sqrt{2})[[X, Y]] = Q(\sqrt{2}) + M$ and $R = Q + M$. By Theorem 3.1 (1), R is a DW-domain, however T is not a DW-domain since it is a Krull domain which is not Dedekind.

2) The assumption of being a "DW-domain" on D is not sufficient to get R a DW-domain when T is not local. Indeed, let $T = Q(\sqrt{2})[X, Y] = Q(\sqrt{2}) + M$ and let $R = Q + M$. Let I be the finitely generated ideal of

R given by $I = (X-1)R + YR$. It is easy to see that $I^{-1} = R$. Then $I_t = I_v = R$. So $I_w = R$. Hence I is not a w-ideal and therefore R is not a DW-domain.

Corollary 3.5. Let K be a field, D a domain contained in K and X an indeterminate over K. Set $R = D + XK[X]$ (respectively $R = D + XK[[X]]$). Then R is a DW-domain if and only if so is D.

Proof. It suffices to take $T = K[X]$ (respectively $T = K[[X]]$).

Theorem 3.6. For the diagram of type (Δ), assume that $qf(D) \subseteq T/M$. If T and D are DW-domains, then so is R.

The proof need the following lemma.

Lemma 3.7. For the diagram (Δ), assume that $D = k$ is a field. If T is a DW-domain, then so is R.

Proof. Let J be a nonzero ideal of R and let $x \in J_w$. Then there exists $A \in GV(R)$ such that $xA \subseteq J$. Since $A^{-1} = R \subset T \subseteq (M : M) \subseteq M^{-1}$, then $A \nsubseteq M$. Since M is a maximal ideal of R ($R/M = D = k$), then $M + A = R$. So there exists $a \in A$ and $m \in M$ such that $1 = a + m$. Hence $x = xa + xm$. Since $xa \in xA \subseteq J$, to show that $x \in J$, it suffices to show that $xm \in J$. Since T is t-linked over R (Lemma 3.2), then $AT \in GV(T)$. But $xA \subseteq J$ implies that $xAT \subseteq JT$ and then $x \in (JT)_{w_1}$, where w_1 is the w-operation with respect to T. Since T is a DW-domain, then $(JT)_{w_1} = JT$ and therefore $x \in JT$. Hence $xm \in JTM = JM \subseteq J$, as desired. It follows that $J = J_w$ and therefore R is a DW-domain.

Proof of the Theorem. Consider the following diagram:

$$\begin{array}{ccc} R & \longrightarrow & D \\ \downarrow & & \downarrow \\ R_0 & \longrightarrow & k \\ \downarrow & & \downarrow \\ T & \longrightarrow & T/M \end{array}$$

By Lemma 3.7, R_0 is a DW-domain and by Theorem 3.1, R is a DW-domain.

Now, we turn our attention to the composite ring $R = A + XB[[X]]$, which is a particular case of the diagram of type Δ, but in which the assumption $T = B[[X]]$ is a DW-domain is not required since this forces B to be a field, Proposition 2.6.

Proposition 3.8 Let $R = A + XB[[X]]$ where $qf(A) \subseteq B$. Then R is a DW-domain if and only if so is A. In particular $k + XB[[X]]$ is always a DW-domain.

Proof. Let $I \in GV(R)$ and set $M = XB[[X]]$. Write $I = (f_1, \ldots, f_r)$, where $f_i = a_i + Xg_i$ for each $i \in \{1, \ldots, r\}$. Since $M^{-1} = (M : M) = B[[X]]$ and $I^{-1} = R$, then $I \not\subseteq M$. So $a_{i_0} \neq 0$ for some $i_0 \in \{1, \ldots, r\}$. Let Ω be the set of all $j \in \{1, \ldots, r\}$ such that $a_j \neq 0$. Clearly Ω is nonempty. Let L be the ideal of A given by $L = \sum_{j\in\Omega} a_j A$ and let J the ideal of R given by $J = L + M$. It is easy to see that $I \subseteq J$. Since $R \subseteq L^{-1} + M = J^{-1} \subseteq I^{-1} = R$, Then $L^{-1} + M = J^{-1} = R$. So $L^{-1} = A$. Hence $L \in GV(A)$. Since A is a DW-domain, then $L = A$. So $1 \in L$. Write $1 = \sum_{j\in\Omega} \lambda_j a_j$. Then $1 + X \sum_{j\in\Omega} \lambda_j g_j = \sum_{j\in\Omega} \lambda_j f_j \in I$. Since $1 + X \sum_{j\in\Omega} \lambda_j g_j$ is a unit of R (recall that $U(R) = U(A) + M$), then $I = R$. It follows that $GV(R) = \{R\}$ and therefore R is a DW-domain.

The following proposition shows that the result of Proposition 3.8 is not true if $qf(A) \not\subseteq B$.

Proposition 3.9. Let $R = A + XB[[X]]$ (respectively $R = A + XB[X]$). If there is a nonzero nonunit $a \in A$ such that $a^{-1}A \cap B = A$, then R is not a DW-domain.

Proof. Set $I = aR + XR$. Then it is easy to show that $I^{-1} = R$. So $I \in GV(R)$. but since a is nonunit of A, then I is a proper ideal of R. It follows that R is not a DW-domain.

Example 3.10. The domain $R = Z + XZ_{(2)}[[X]]$, where Z is the ring of integers is not a DW-domain. Indeed 2 is nonunit in Z and $2^{-1}Z \cap Z_{(2)} = Z$.

Reference

[1] D.F. Anderson and D.E. Dobbs, Pairs of rings with the same prime ideals, Canad. J. Math. 32 (1980), 362–384.

[2] E. Bastida and R. Gilmer, Overrings and divisorial ideals of rings of the form D+M, Michigan Math. J. 20 (1973), 79–95.

[3] D. Dobbs and F. Fedder, Conducive integral domains, J. Algebra 86 (1984), 494–510.

[4] M. Fontana, S.Gabelli and E. Houston, UMT-domain and domains with Prüfer integral closure, Comm. Algebra 26 (4)(1998), 1017–1039.

[5] M. Fontana, J. Huckaba, and I. Papick, Prüfer domains, M. Dekker, New York, 1997.

[6] S.Gabelli and E. Houston Coherent like conditions in Pullbacks, Michigan Math. J. 44 (1997), 99–112.

[7] Gilmer, Multiplicative Ideal Theory, Marcel Dekker, Inc., New York, 1972.

[8] W. Heinzer, Integral domains in which each non-zero ideal is divisorial, Mathematika 15 (1990), 164–170.

[9] E. Houston and M. Zafrullah, Integral domains in which each t-ideal is divisorial, Michigan Math. J. 35 (1988), 291–300.

[10] I. Kaplansky, Commutative rings, Rev. ed. Univ. Chicago Press, 1974.

[11] Mi He Park, group rings and semigroup rings over strong Mori domains, J. Pure Appl. Algebra, 163 (2001), 301–318.

[12] A. Mimouni, TW-domains and strong Mori domains, J.Pure Appl. Algebra 177 (2003), 79–93.

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