A Factorization Theorem for Haken Manifolds

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Abstract

In this article, we deal with a map $f: M \longrightarrow N$ between two closed Haken manifolds such that each finite covering of f is a homological equivalence with integer coefficients. The main result says that if we control f on the non-degenerate hyperbolic part of M then f can be homotoped to a collapse between the Seifert part of M and that of N. As a consequence, we show that the Seifert part of N is obtained from that of M by a finite number of Dehn fillings.

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1 Introduction

1.1 The main result

In this paper we deal with a map $f : M \longrightarrow N$ between two closed Haken manifolds such that each finite covering of f is a $\mathbf{Z}$-homology equivalence. For each closed Haken manifold N we denote by $\mathcal{T}(N)$ the family of Jaco-Shalen-Johannson characteristical tori and by $\mathcal{H}(N)$ (resp. $\mathcal{S}(N)$) the disjoint union of hyperbolic pieces (resp. Seifert pieces) of N (see [7], [8], [16] and [18]). We equip the space $\mathcal{H}(N)$ (resp. $\mathcal{S}(N)$) with the following equivalence relation: Let x, y be two points of $\mathcal{H}(N)$ (resp. of $\mathcal{S}(N)$). Then $x \sim y$ if and only if there are two tori U and V of $\partial\mathcal{H}(N)$ (resp. of $\partial\mathcal{S}(N)$) such that $U \ni x$ and $V \ni y$, and a homeomorphism $\varphi : U \longrightarrow V$, which identifies U with V in N, such that $\varphi(x) = y$. Let H_N (resp. S_N) denotes the space $\mathcal{H}(N)/\sim$ (resp. $\mathcal{S}(N)/\sim$). If $f : M \longrightarrow N$ is a map between two Haken manifolds we denote by $\mathcal{H}^+(M)$ (resp. $\mathcal{H}^-(M)$) the disjoint union made of components Q of $\mathcal{H}(M)$ such that $f_*(\pi_1(Q))$ is a non-cyclic group (resp. cyclic group).

In this article, the goal of the main result is to show that if we control f on the components of $\mathcal{H}^+(M)$ then the Seifert part of N is obtained from that of M by a finite number of Dehn fillings on the components of $\mathcal{S}(M)$. To state precisely this result, we need the following definition:

Definition 1.1 *Let M be a compact, closed three-manifold and let $\mathcal{Q}$ be a codimension zero submanifold of M whose components have connected boundary homeomorphic to a torus. We say that M collapses along $\mathcal{Q}$ if for each component $Q_1, \ldots, Q_n$ of $\mathcal{Q}$ there is a homeomorphism $\varphi_i : \partial(\mathcal{D}^2 \times \mathbf{S}^1) \longrightarrow \partial A_i = \partial(\overline{M - A_i})$ and a map*

$$\pi : M \longrightarrow M_1 = (\overline{M - \mathcal{Q}}) \cup_{\varphi_1} (\mathcal{D}^2 \times \mathbf{S}^1) \cup \ldots \cup_{\varphi_n} (\mathcal{D}^2 \times \mathbf{S}^1)$$

$$\textit{such that} \quad \pi | \overline{M - \mathcal{Q}} = id \quad \textit{and} \quad \pi(Q_i) = \mathcal{D}^2 \times \mathbf{S}^1.$$

Then we say that M_1 is a collapse of M and π is the associated collapsing projection.

The main result of this paper states as follows:

Theorem 1.2 *Let $f : M \longrightarrow N$ be a map between two closed Haken manifolds such that each finite covering of f is a $\mathbf{Z}$-homology equivalence. If $f(\mathcal{H}(M)^+) \subset H_N$, there exists a submanifold $\mathcal{Q}$ of M, a collapse $\pi : M \longrightarrow M_1$ of M along Q and a map $f_1 : M_1 \longrightarrow N$ inducing a homeomorphism between $(\mathcal{S}(M_1), \partial\mathcal{S}(M_1))$ and $(\mathcal{S}(N), \partial\mathcal{S}(N))$ such that f is homotopic to $f_1 \circ \pi$. Moreover $(\mathcal{S}(M_1), \partial\mathcal{S}(M_1))$ is obtained from $(\mathcal{S}(M), \partial\mathcal{S}(M))$ by Dehn fillings.*

1.2 The motivation

In [12], B. Perron and P. Shalen show that any map between two irreducible closed graph manifolds with infinite π_1 whose each finite covering is a $\mathbf{Z}$-homology equivalence is homotopic to a homeomorphism. In [3] we extend this result for closed Haken manifolds with the same Gromov simplicial volume denoted by $\|.\|$ (see [4, paragraph 0.2] or [17, paragraph 6.1] for definitions). More precisely, we show that any map between two closed Haken manifolds with the same Gromov simplicial volume whose each finite covering is a $\mathbf{Z}$-homology equivalence is homotopic to a homeomorphism. The hypothesis concerning the Gromov simplicial volume of the given manifolds is necessary for [3, Theorem 1]. Indeed, if $\|M\| = \|N\|$ then we know the behavior of f on the hyperbolic part of M, since this hypothesis allows us to show that f is homotopic to a map g such that $g(\mathcal{H}(M), \partial\mathcal{H}(M)) \subset (\mathcal{H}(N), \partial\mathcal{H}(N))$. However, without this hypothesis, we may construct some conter example to [3, Theorem 1] (see examples of section 2).

However, this rigidity result raises the following natural question: understand, up to homotopy, the behavior of the map f without hypothesis on the Gromov simplicial volume of the given manifolds. We want to know if there is a “nice" relation between the Seifert part of M and that of N. Generally, there is no relation (see example 1 or example 4). However, these two parts are closely related adding a necessary hypothesis on the behavior of $f|\mathcal{H}(M)$. In fact, it is necessary to control the images of the hyperbolic submanifolds H of M for which $f_*(\pi_1(H))$ is a non cyclic-group (see examples of section 2). The main reason which justifies this “control" is that any Haken manifold can be seen as the image of a hyperbolic 3-manifold by a degree one map. Moreover, this map can be chosen in such a way that all its finite coverings are **Z**-homology equivalences. In the proof of Theorem 1.2 we need the following result:

Proposition 1.3 *Let $f : M \longrightarrow N$ be a map between two closed Haken manifolds satisfying the hypothesis of Theorem 1.2. If, for each component H of $\mathcal{H}(M)$ $f_*(\pi_1(H))$ is a non cyclic group, then f is homotopic to a map which induces a homeomorphism between $(\mathcal{S}(M), \partial\mathcal{S}(M))$ and $(\mathcal{S}(N), \partial\mathcal{S}(N))$.*

In particular, we may show the following fact: Let M, N be two closed Haken manifolds and let $f : M \longrightarrow N$ be a map such that each finite covering of f is a **Z**-homology equivalence. If each component of $\mathcal{H}(M)$ degenerates (i.e. if $\mathcal{H}(M) = \mathcal{H}^-(M)$) then N is a graph manifold. The converse is obviously false (see Example 1). Then we have the following corollary which approximatively says that when the hyperbolic pieces of M degenerate we can control their images:

Corollary 1.4 *Let $f : M \longrightarrow N$ be a map between closed Haken manifolds such that each finite covering of f is a **Z**-homology equivalence. If $\mathcal{H}(M) = \mathcal{H}^-(M)$, there exists a submanifold $\mathcal{Q}$ of M, a collapse $\pi: M \longrightarrow M_1$ of M along Q and a map $f_1 : M_1 \longrightarrow N$ inducing a homeomorphism between $(\mathcal{S}(M_1), \partial\mathcal{S}(M_1))$ and $(\mathcal{S}(N), \partial\mathcal{S}(N))$ such that $f \simeq f_1 \circ \pi$. Moreover $(\mathcal{S}(M_1), \partial\mathcal{S}(M_1))$ is obtained from $(\mathcal{S}(M), \partial\mathcal{S}(M))$ by Dehn fillings.*

We recall the following definition of a *degenerate map* in the sense of W. Jaco and P. B. Shalen:

Definition 1.5 *Let S be a Seifert fibered space and let N be a closed Haken manifold. We say that a map $f : S \longrightarrow N$ is degenerate if it satisfies at least one of the following conditions:*

(i) the group $Im(f_* : \pi_1(S) \longrightarrow \pi_1(N))$ is cyclic or,
(ii) there exists a fiber γ of S such that the map $f|\gamma$ is homotopic to a constant map in N.

In the same way, we say that a map $f : T \longrightarrow N$ from a torus T to a closed Haken manifold is degenerate if the induced homomorphism $f_* : \pi_1(T) \longrightarrow \pi_1(N)$ is not π_1-injective.

This paper is organised as follows: In section 2 we construct some suitable examples justifying the hypothesis of our results and showing that the conclusions are optimal. In section 3 we state a key result for the proof of the main result. The section 4 is devoted to the proof of Proposition 1.3 and in section 5 we give a proof of the main result (Theorem 1.2) by showing that if we perform a “good" factorization of the map f we have a reduction to the hypothesis of Proposition 1.3.

2 Some examples

In this section we give some examples which justify hypothesis of Theorems 1.2 and 1.3 and which show that the situation is very different when nothing is assumed on the Gromov simplicial volume of the manifolds M and N. Let A and B be two oriented irreducible compact 3-manifolds with boundary a torus. In addition assume that B is not homeomorphic to a solid torus (i.e. ∂B is incompressible). We denote by M_φ the closed manifold obtained by gluing A and B by a homeomorphism $\varphi : \partial A \longrightarrow \partial B$. The following lemma will be usefull for the construction of our examples.

Lemma 2.1 *If each finite cyclic covering $\tilde{A}$ of A satisfies $H_1(\tilde{A}, \mathbf{Z}) = \mathbf{Z}$ then there exists a closed irreducible manifold N_φ obtained by collapsing M_φ along A such that each finite covering of the collapsing projection $\pi : M_\varphi \longrightarrow N_\varphi$ is a $\mathbf{Z}$-homology equivalence.*

Proof: Since $H_1(A, \mathbf{Z}) = \mathbf{Z}$ then there is an epimorphism $h : \pi_1(A) \longrightarrow \mathbf{Z} = \pi_1(V)$ where V denotes a solid torus. Since the spaces V and A are both $K(\pi, 1)$, it follows from obstruction theory that h

is induced by a continuous map $\pi_0 : A \longrightarrow V$. We now show that we may choose π_0 in its homotopy class in such a way that it satisfies the following properties:

(i) $\pi_0 : (A, \partial A) \longrightarrow (V, \partial V)$,

(ii) π_0 induces a homeomorphism $\pi_0|\partial A : \partial A \longrightarrow \partial V$.

First of all note that $h|\pi_1(\partial A) : \pi_1(\partial A) \longrightarrow \pi_1(V)$ is onto. Indeed, the image of h is $H_1(A, \mathbf{Z})$ and since $H_1(A, \mathbf{Z}) = \mathbf{Z}$, the homomorphism induced by the inclusion $H_1(\partial A, \mathbf{Z}) \hookrightarrow H_1(A, \mathbf{Z})$ is onto. This allows to find a basis (λ, ν) of $\pi_1(\partial A)$, represented by two simple closed curves in ∂A, still denoted by (λ, ν), such that $\pi_0(\nu) = l_V$, where l_V denotes the core of V and $\pi_0(\lambda) = m$, where m denotes the meridian of V. So we have defined a map $\pi_0 : \partial A \longrightarrow \partial V$ inducing an isomorphism on fundamental groups. We can now extend this map to a new map $\pi_0 : (A, \partial A) \longrightarrow (V, \partial V)$ satisfying points (i) and (ii) above using the asphericity of V.

We now construct the manifold N_φ and the collapsing projection $\pi : M_\varphi \longrightarrow N_\varphi$. Attach a solid torus V to B along $T = \partial A$ in such a way that the meridian of V is identified with λ and the core l_V of V with ν and denote by N_φ the resulting manifold. Define now the map $\pi_0' : B \longrightarrow N_\varphi \setminus V$ to be the identity and the map $\pi : M_\varphi = A \cup_T B \longrightarrow N_\varphi = V \cup B$ setting $\pi|A = \pi_0$ and $\pi|B = \pi_0'$. It follows from the construction above that π is a well defined and continuous map. It remains to show that each finite covering of π is a $\mathbf{Z}$-homology equivalence. Let $q : \tilde{N}_\varphi \longrightarrow N_\varphi$ be the finite covering of N_φ, $p : \tilde{M}_\varphi \longrightarrow M_\varphi$ be the (finite) covering induced by π over M and let $\tilde{\pi} : \tilde{M} \longrightarrow \tilde{N}$ be a lifting of π. Since the images of the homomorphisms induced by inclusions $i_* : \pi_1(V) \longrightarrow \pi_1(N_\varphi)$ and $j_* : \pi_1(\partial V) \longrightarrow \pi_1(N_\varphi)$ are equal in $\pi_1(N_\varphi)$ then the connected components of $q^{-1}(V)$ have all connected boundary, homeomomorphic to a torus and then $\tilde{B} = q^{-1}(B)$ is connected. Denote by $V_1, \ldots, V_k$ the connected components of $q^{-1}(V)$. By the same argument, since $\pi_*(\pi_1(A)) = \pi_*(\pi_1(\partial A)) = \mathbf{Z}$, the components $A_1, \ldots, A_l$ of $p^{-1}(A)$ have all connected boundary and since $\pi|\partial A : \partial A \longrightarrow \partial V$ is a homeomorphism then $l = k$. Moreover, since the coverings $A_1, \ldots, A_l$ are (finite) cyclic then $H_1(A_1, \mathbf{Z}) = \ldots = H_1(A_k, \mathbf{Z}) = \mathbf{Z}$. So the map $\tilde{\pi}$ satisfies $\tilde{\pi}(A_i, \partial A_i) = (V_i, \partial V_i)$ where $\tilde{\pi}|\partial A_i : \partial A_i \longrightarrow \partial V_i$ is a homeomorphism for each i and $\tilde{\pi}|\tilde{B} : \tilde{B} \longrightarrow \tilde{B}$ is the identity. Using the exact sequence of pairs $(A_i, \partial A_i)$ and $(V_i, \partial V_i)$ and Poincaré's duality it is easy to show that for each i, the map $\pi_i = \tilde{\pi}|(A_i, \partial A_i) : (A_i, \partial A_i) \longrightarrow (V_i, \partial V_i)$ is a $\mathbf{Z}$-homology equivalence. Finally we can prove that $\tilde{\pi}$ is a $\mathbf{Z}$-homology equivalence by a standard Mayer-Vietoris argument. This completes the proof of Lemma 2.1. ■

Example 1: The goal of this example is to show that the control of the non-degenerate hyperbolic part of M is necessary for Theorem 1.2 (see figure 1).

Combining the main result of [9] or [19] (see also [13, Theorem p. 273]) with a result of Myers (see [10] and [11]) which says that if M is an orientable 3-manifold which contains no 2-sphere in its boundary then any simple closed curve γ is properly homotopic to a curve γ' such that $\overline{M - N(\gamma')}$ is hyperbolic, we can show that each compact 3-manifold may be obtained from the complement of a hyperbolic link up to a finite number of Dehn fillings (see also [1, Corollary E.7.10]). Let N be a closed graph manifold and denote by $l_1, \ldots, l_k$ the link which generates N by Dehn fillings. Set $B = \mathbf{S}^3 \setminus \{l_1, \ldots, l_k\}$. For each $i = 1, \ldots, k$, we denote by T_i the torus of ∂B corresponding to l_i. Then we can identify N to $B \cup_{T_1} V_1 \cup \ldots \cup_{T_k} V_k$, where each V_i is a solid torus. For each i, we denote by m_i the curve of T_i identified to the meridian of V_i.

Figure 1: Hyperbolic manifolds collapse to a graph manifold

We now choose k hyperbolic knots whose complements are denoted by $H_1, \ldots, H_k$ and whose each cyclic finite covering has their first homology group isomorphic to $\mathbf{Z}$. This choice is clearly possible, indeed, it is sufficient to take hyperbolic knots whose Alexander polynomial is trivial and then apply the Fox formula (see [20]). For each $i \in \{1, \ldots, k\}$, we glue H_i to B along T_i identifying the meridian of H_i to m_i and then we apply Lemma 2.1.

So we obtain a Haken manifold M, whose geometric pieces are all hyperbolic, a graph manifold N and a collapsing map $\pi : M \longrightarrow N$ such that each finite covering of π is a $\mathbf{Z}$-homology equivalence.

Example 2: The goal of this example is to show that without hypothesis on the Gromov simplicial volume on the given manifold, some canonical tori of M can degenerate. Let S be a Seifert fibered space with two boundary components T_1 and T_2 and with at least two exceptional fibers. Along T_2 we glue a Haken manifold B whose boundary is made of a single torus and along T_1 we glue the complement of a hyperbolic knot A whose Alexander polynomial is trivial by identifying the meridian of A with a simple closed curve which is not a fiber of S in T_1. So we may construct a Haken manifold N obtained from $M = A \cup_{T_1} S \cup_{T_2} B$ by a collapse along A. The projection $\pi : M \longrightarrow N$ corresponding to the collapse is a map whose each finite covering is a $\mathbf{Z}$-homology equivalence and the canonical torus T_1 of M degenerates.

Example 3: We construct here a map f between two closed Haken manifolds M and N, satisfying the hypothesis of Theorem 1.2, such that some Seifert fibered components of M degenerate under f (see figure 2).

Figure 2: Degeneration of the Seifert part of M

We first fix two hyperbolic knots, whose complements are denoted by H_1 and H_2 and whose Alexander polynomials are trivial, and denote by m_1 and m_2 the respective meridian of H_1 and H_2. Let S be a Seifert fibered space obtained on the following way: We consider a 2-sphere with three holes, denoted by F and we set $S = F \times \mathbf{S}^1$. Recall that the fundamental group of S has a presentation:

$$\pi_1(S) = \langle d_1, d_2, d_3 |\ d_3 = d_1.d_2 \rangle \times \langle h \rangle$$

where the d_i's correspond to the boundary components of F and where h corresponds to $\mathbf{S}^1$. We now choose a hyperbolic manifold H_3 with two boundary components denoted by T_3 and T_4 and a Haken manifold B whose boundary is made of a torus T. We choose a simple closed curve m_3, in T_3, in such a way that if we perfom a Dehn filling on H_3 along m_3, we still obtain a complete finite volume hyperbolic manifold. Let M_1 be the manifold obtained in the following way:

$$M_1 = H_1 \bigcup_{m_1 = d_1} S \bigcup_{m_2 = d_2} H_2$$

It is now easy to define a map $\pi : (M_1, \partial M_1) \longrightarrow (V, \partial V)$ such that each finite covering of π is a **Z**-homology equivalence and which induces a homeomorphism on the boundary. More precisely, $\pi(h)$ is the core of V and $\pi(d_3)$ is the meridian. Let M_2 be the manifold obtained by gluing H_3 and B together, along T by an orientation preserving homeomorphism $\varphi : T_4 \longrightarrow T$. Finally let M be the manifold obtained from M_1 and M_2 identifying ∂M_1 and ∂M_2 by $d_3 = m_3$ and let N be the Haken manifold obtained from M_2 glueing a solid torus along T_3 by identifying its meridian to m_3. So we may define a map $f : M \longrightarrow N$ such that each finite covering of f is a **Z**-homology equivalence and such that $f_*(\pi_1(S)) \simeq \mathbf{Z}$.

<u>Example 4</u>: In this example, we show that the control of the hyperbolic part of M is necessary for the conclusion of Proposition 1.3 even if the canonical tori of M do not degenerate. Let B be a Haken manifold whose boundary is made of a single torus and whose hyperbolic and Seifert part is non-empty. Let S be an orientable, irreducible Seifert fibered manifold with connected boundary. Let N be the manifold $B \cup S$ where ∂B is identified to ∂S by an orientation preserving homeomorphism. Our construction is based on the following claim whose proof may be found in [2]:

Claim 1 *([2]): We may choose a totally nul-homotopic knot k in* $\mathrm{int}(B)$ *such that the manifold $\overline{B - N(k)}$ is simple, i.e. $\overline{B - N(k)}$ is irreducible, ∂-irreducible and contains no essential torus or annulus.*

It follows from Thurston's hyperbolization Theorem that $\overline{B - N(k)}$ is a Haken hyperbolic manifold. In the following we denote by m the meridian of $N(k)$. Since k is nul-homotopic, there exists a singular disk D in $\mathrm{int}(B)$ such that $k = \partial D$ and D is homotopically equivalent to a graph. Let $N(D)$ be a regular neighbourhood of D sufficiently large in such a way that $N(D) \supset N(k)$. Thus $N(D)$ is an irreducible handleboddy. Let $\alpha = 1.m + a.k$ be a primitive curve in $\partial N(K)$ with $a \neq 0$. We denote by $B(k, a)$ the manifold obtained from $\overline{B - N(k)}$ after performing a Dehn filling along $\partial N(k)$ by identifying the curve α to the meridian. It follows from Thurston's hyperbolic surgery Theorem that if a is sufficiently large then the manifold $Q = B(k, a)$ supports a complete finite volume hyperbolic structure. Since α is homotopic to a multiple of k in $N(k)$ then α is homotopic to zero in $N(D)$. Then we may define a degree one map:

$$f : Q = B(k, a) = \overline{B - N(k)} \cup D^2 \times S^1 \longrightarrow \overline{B - N(k)} \cup N(k) = B$$

in the following way: We set $f|\overline{B - N(k)} : \overline{B - N(k)} \longrightarrow \overline{B - N(k)}$ to be the identity. Since α is homotopically trivial in $N(D)$, we extend

$f|\overline{B-N(k)}$ to $\overline{B-N(k)} \cup (D^2 \times *)$ mapping $(D^2 \times *)$ to $N(D)$, where $* \in S^1$ and $\partial D^2 \times *$ is α. Now $\overline{D^2 \times S^1 - D^2 \times *}$ is a 3-disk D^3. Since $N(D)$ is irreducible and since $f(\partial D^3) \subset N(D)$, we may extend this map to the whole manifold Q by sending D^3 in $N(D)$. So we obtain a hyperbolic 3-manifold Q and a degree one map $f : (Q, \partial Q) \longrightarrow (B, \partial B)$ such that $f|\partial Q : \partial Q \longrightarrow \partial B$ is a homeomorphism. Moreover, since the knot k is totally nul-homotopic we can show that each finite covering of f is a $\mathbf{Z}$-homology equivalence. We can now extend f to a map, still denoted by f, between $M = Q \cup S$ and N with the same property. This completes the construction of Example 4 and shows that the Seifert part of N is not obtained from that of M by Dehn fillings.

Example 5: In this last example, we show that the control of the map f along the hyperbolic manifolds whose fundamental group is mapped to a non-cyclic abelian group is necessary. Let S be the space $\mathbf{S}^1 \times \mathbf{S}^1 \times I$ where I denotes the interval $[0,1]$, (m^+, l^+) (resp. (m^-, l^-)) a basis of "meridian-parallel" of $\mathbf{S}^1 \times \mathbf{S}^1 \times \{+1\}$ (resp. $\mathbf{S}^1 \times \mathbf{S}^1 \times \{-1\}$). Using the construction of Example 4, we can construct a complete finite volume hyperbolic manifold H with two boundary components (homeomorphic to tori) and a map $\pi : (H, \partial H) \longrightarrow (S, \partial S)$ which induces a homeomorphism on the boundary and such that each finite covering of π is a $\mathbf{Z}$-homology equivalence. Denote by (m_H^+, l_H^+) (resp. (m_H^-, l_H^-)) a basis of $\pi^{-1}(\mathbf{S}^1 \times \mathbf{S}^1 \times \{+1\}) = T^+$ (resp. $\pi^{-1}(\mathbf{S}^1 \times \mathbf{S}^1 \times \{-1\}) = T^-$) such that $\pi(m_H^+) = m^+, \pi(l_H^+) = l^+$ (resp. $\pi(m_H^-) = m^-, \pi(l_H^-) = l^-$). Then we choose two Seifert fibered spaces $S^+ = F^+ \times \mathbf{S}^1$, $S^- = F^- \times \mathbf{S}^1$, where F^+ and F^- are orientable surfaces with connected boundary. Recall that S^+ and S^- have presentations:

$$\pi_1(S^+) = \left\langle a_1, b_1, \ldots, a_g, b_g, d^+ | \ d^+ \left(\prod [a_i, b_i]\right) = 1 \right\rangle \times \langle t \rangle$$

$$\pi_1(S^-) = \left\langle a_1, b_1, \ldots, a_{g'}, b_{g'}, d^- | \ d^- \left(\prod [a_i, b_i]\right) = 1 \right\rangle \times \langle t \rangle$$

Let N (resp. M) be the space obtained from $S^+ \bigcup (\mathbf{S}^1 \times \mathbf{S}^1 \times I) \bigcup S^-$ (resp. $S^+ \bigcup H \bigcup S^-$) identifying m^+ to d^+, l^+ to t, m^- to d^-, l^- to t and m_H^+ to d^+, l_H^+ to t, m_H^- to d^-, l_H^- to t. Since in $\mathbf{S}^1 \times \mathbf{S}^1 \times I$ the elements l^+ and l^- are conjugated and since they are identified to the regular fiber of S^+ and of S^- then the Seifert fibration of S^+ extends to the Seifert fibration of N and the manifold M is a Haken manifold whose Gromov simplicial volume is non zero and is (geometrically) made of two Seifert fibered manifolds and one hyperbolic manifold for the Jaco-Shalen-Johannson decomposition. Moreover, the map π built at the beginning extends immediatly (to the identity) to a new map, still denoted by $\pi : M \longrightarrow N$, such that each finite covering of π is a $\mathbf{Z}$-homology equivalence. This completes the construction of Example 5 since $\mathcal{S}(N)$ is not a collapse of $\mathcal{S}(M)$.

3 Ends of M

In this section we deal with a map $f : M \longrightarrow N$ between closed Haken manifolds such that each finite covering of f is a $\mathbf{Z}$-homology equivalence. We always assume that M has a non-trivial torus decomposition (i.e. $\mathcal{T}(M) \neq \emptyset$). Then we state here a key result for our proof of Theorem 1.2.

Lemma 3.1 *If T is a degenerate canonical torus of M, there exists a unique component A of $M \setminus T$ such that $H_1(A, \mathbf{Z}) = \mathbf{Z}$ and $f_*(\pi_1(A))$ is a cyclic group. Moreover this property is invariant by finite coverings over A induced by the finite coverings of N via $f|A$.*

In the following, we call *end of M* any codimension zero submanifold of M whose boundary is made of a single canonical torus of M and satisfying the properties of the component A of Lemma 3.1. The proof of the result above depends on the following claim which appears in [3, Lemma 4.1].

Claim 2 *If T denotes a degenerate canonical torus in M then T separates M into two submanifolds A and B and there is a component, say A, of $M - T$ satisfying the following property: for any finite covering p of M induced by f from some finite covering of N then each component $\tilde{A}$ of $p^{-1}(A)$ satisfies $H_1(A, \mathbf{Z}) = \mathbf{Z}$ (in particular $\partial\tilde{A}$ is connected).*

The proof of Claim 2 uses arguments quite similar to those of the proof of Lemma 4.1 in [3]. More precisely, in the present case, the component A may contain some hyperbolic pieces of $\mathcal{H}(M)$ whereas in [3, Lemma 4.1] A is a graph submanifold of M. Since the proof of Lemma 4.1 in [3] does not use the geometric structure of A and B, this proof works also in the present case without any essential change.

Proof of Lemma 3.1: According to Claim 2, to prove the lemma we first show that $G = f_*(\pi_1(A))$ is an abelian group. Suppose the contrary and denote by H the group $f_*(\pi_1(\partial A)) < G < \pi_1(N)$. Then we show that there exists a finite covering of M in which A lifts to a space whose components have disconnected boundary which contradicts Claim 2.

Case 1: Assume that H is trivial. Since G is non-abelian, we may choose an element $g \in G - H$. Since $\pi_1(N)$ is a residually finite group, there exists a finite group K and an epimorphism $\varphi : \pi_1(N) \longrightarrow K$ such that $\varphi(g) \neq 1$. Denote by $\tilde{A}$ a connected component of the finite covering of A corresponding to $\varphi \circ (f|A)_*$. It follows from the construction above, that $\partial\tilde{A}$ has at least two boundary components. A contradiction.

Case 2: Assume that H is infinite cyclic. Let h be an element of $\pi_1(\partial A)$ such that $f_*(h)$ generates H. Since G is non-abelian, there exists elements a and b in $\pi_1(A)$ such that $[f_*(a), f_*(b)] \neq 1$. Since $\pi_1(N)$ is a residually finite group, there exists a finite group K and an epimorphism $\varphi : \pi_1(N) \longrightarrow K$ such that $[\varphi f_*(a), \varphi f_*(b)] \neq 1$. Let $\tilde{A}$ be a connected component of the finite covering of A corresponding to $\varphi \circ (f|A)_*$ and set $K' = \varphi \circ (f|A)_*(\pi_1(A))$. Then clearly $\partial\tilde{A}$ is disconnected. Indeed, $\partial\tilde{A}$ is connected if and only if $\langle \varphi f_*(h) \rangle = K'$. But this would imply that K' is abelian which is impossible since $[\varphi f_*(a), \varphi f_*(b)] \neq 1$. A contradiction.

So G is a torsion free abelian goup and thus $(f|A)_*$ factors throught $H_1(A, \mathbf{Z}) = \mathbf{Z}$ and since $\pi_1(N)$ is a torsion free group we get $f_*(\pi_1(A)) = 1$ or $\mathbf{Z}$.

We now assume that for each finite covering of M, each component $\tilde{A}$ over A and $\tilde{B}$ over B satisfies $H_1(\tilde{A}, \mathbf{Z}) = H_1(\tilde{B}, \mathbf{Z}) = \mathbf{Z}$. Applying the same argument as above, we can show that necessarily, $f_*(\pi_1(A)) \simeq 1$ or $\mathbf{Z}$ and $f_*(\pi_1(B)) \simeq 1$ or $\mathbf{Z}$, which implies using the Van-Kampen Theorem for the decomposition $M = A \cup B$ and since f is a degree one map, that $\pi_1(N) = 1$ or $\mathbf{Z}$. Since N is a Haken manifold, this shows that N is either a 3-sphere $\mathbf{S}^3$ or a solid torus, which is impossible since the given manifold are chosen closed. This proves the unicity of the component A and completes the proof of Lemma 3.1.

4 Proof of Proposition 1.3

In this section we deal with a map $f : M \longrightarrow N$ between closed Haken manifolds such that each finite covering of f is a $\mathbf{Z}$-homology equivalence. We always assume that f satisfies hypothesis of Proposition 1.3. That means that $\mathcal{H}(M) = \mathcal{H}^+(M)$ and $f(\mathcal{H}^+(M)) \subset H_N$. Moreover, we may assume, in view of the main result of B. Perron and P. Shalen, that M is not a graph manifold (otherwise, N would be a graph manifold too). Thus necessarily, $H_M \neq \emptyset$.

4.1 Preliminaries

Lemma 4.1 *For each canonical torus T of $\mathcal{T}(M)$, the map $f|T$ is π_1-injective.*

Let T be a canonical torus of M which degenerates under f. We know by Lemma 3.1, that T separates M into two submanifolds A and B and that one component of $M - T$, says A, satisfies $H_1(A, \mathbf{Z}) = \mathbf{Z}$ and $f_*(\pi_1(A)) = 1$ or $\mathbf{Z}$. Recall that if M is a 3-manifold, T a component of ∂M homeomorphic to a torus, and if (m, l) is a basis of meridian-parallel of T, we call *Dehn filling* along T with coefficient (a, b) the operation which consists of glueing a solid torus $\mathcal{D}^2 \times \mathbf{S}^1$ to M along T by identifying $am + bl$ to $\partial\mathcal{D}^2 \times \{*\}$, where $* \in \mathbf{S}^1$. The proof of Lemma 4.1 depends on the following result:

Lemma 4.2 *Let A be an end of M and let Q be a geometrical component of M adajacent to A along $T = \partial A$. If $f|Q : Q \longrightarrow N$ is a non-degenerate map then $f_*(\pi_1(A)) \neq 1$.*

Proof: If Q is a Seifert fibered manifold then it is non-degenerate in the sense of Jaco and Shalen. In particular, if h denotes the regular fiber of Q then $f_*(h) \neq 1$. Since h is represented in $T = \partial A \subset \partial Q$ then $f_*(\pi_1(A)) = f_*(\pi_1(\partial A)) = f_*(\pi_1(T)) = \mathbf{Z}$. Assume now that Q is a hyperbolic piece of M and that $f_*(\pi_1(A)) = f_*(\pi_1(T)) = \{1\}$. Denote by $T_1, \ldots, T_k$ the other components of ∂Q. Let (m, l) (resp. (m_i, l_i), $i = 1, \ldots, k$) be a basis of meridian parallel for T (resp. for T_i, $i = 1, \ldots, k$). It follows from Thurston's hyperbolic surgery Theorem that there exists an element q of $\mathbf{R}^2$ such that any Dehn filling of Q along T, $T_1, \ldots, T_k$ with coefficients (a, b), (a_i, b_i), $i = 1, \ldots, k$ with $\|(a, b)\|$ and $\|(a_i, b_i)\| \geq \|q\|$ supports a complete finite volume hyperbolic structure. Let $\hat{Q}$ be the manifold $A \cup_T Q$. Since $f_*(\pi_1(A)) = \{1\}$ then $f|\hat{Q} : \hat{Q} \longrightarrow N$ factors throught any manifold $\hat{Q}_{(a,b)}$ obtained from Q by Dehn filling along T with coefficient (a, b) ($\|(a, b)\| \geq \|q\|$), when $(a, b) \in \mathbf{Z}^2$. Let $\pi_{(a,b)} : \hat{Q} \longrightarrow \hat{Q}_{(a,b)}$ be the collapsing map and $f' : \hat{Q}_{(a,b)} \longrightarrow N$ be the map such that $f|\hat{Q} = f' \circ \pi_{(a,b)}$. Clearly, $\pi_{(a,b)}$ is a degree one map and $\hat{Q}_{(a,b)}$ is a complete finite volume hyperbolic manifold. Using the same arguments as in proof of Lemma 2.1, we show that we can extend $\pi_{(a,b)}$ to a degree one map from $\hat{Q}_{((a_1,b_1),\ldots,(a_k,b_k))}$ to $\hat{Q}_{((a,b),(a_1,b_1),\ldots,(a_k,b_k))}$ still denoted by $\pi_{(a,b)}$, where the $(a_i, b_i)'$s are chosen such that $\|(a_i, b_i)\| \geq \|q\|$ and will be fixed in the following and where $\hat{Q}_{((a_1,b_1),\ldots,(a_k,b_k))}$ (resp. $\hat{Q}_{((a,b),(a_1,b_1),\ldots,(a_k,b_k))}$) denotes the manifold obtained from $\hat{Q}$ (resp. of Q)

by Dehn filling along T_i (resp. along T and T_i). It follows from the construction above that the manifolds $\hat{Q}_{((a,b),(a_1,b_1),\ldots,(a_k,b_k))}$ support a complete finite volume hyperbolic structure for all $\|(a,b)\| \geq \|q\|$ and these manifolds are pairwise non-homeomorphic. Thus we have defined an infinite sequence of closed hyperbolic manifolds dominated, via the sequence of maps $\{\pi_{(a,b)}\}_{\{\|(a,b)\|\geq q\}}$, by $\hat{Q}_{((a_1,b_1),\ldots,(a_k,b_k))}$ which contradicts [15, Theorem 1]. This proves that $f_*(\pi_1(A)) = \mathbf{Z}$. ∎

Proof of Lemma 4.1: Suppose the contrary. Then there exists a canonical torus T in M such that $f|T$ is a degenerate map. Since the hyperbolic pieces of M are non-degenerate ($H_M \neq \emptyset$), we may choose an end A of M adjacent to a non-degenerate piece of M. Thus it follows from Lemma 4.2 that $f_*(\pi_1(A)) = \mathbf{Z}$. Since each canonical torus of A degenerates, the hypothesis of Proposition 1.3 implies that A is a graph manifold. By [3, lemme 5.1] we know that there is a component S of A which admits a Seifert fibration whose orbit space is a 2-disk (with at least two exceptional fibers) and satisfying $f_*(\pi_1(S)) \simeq \mathbf{Z}$. By [5, Theorem 1] we know that for each pair of elements g, h of $f_*(\pi_1(S))$ such that $g \notin \langle h \rangle$, there exists an epimorphism $\varphi : \pi_1(N) \longrightarrow K$, where K is a finite group, such that $\varphi(g) \notin \langle \varphi(h) \rangle$. Let $c_1, \ldots, c_r$ be the homotopy class of the exceptional fibers and let h be the homotopy class of the regular fiber of S. Note that $r \geq 2$, otherwise S would be homeomorphic to a solid torus which is impossible since the canonical tori of M are incompressible. Thus we have, for $i = 1, \ldots, r$, $c_i^{\mu_i} = h$, where $\mu_i > 1$ for all i. Since $\pi_1(N)$ is a torsion free group then necessarily, $f_*(c_i) \notin \langle f_*(h) \rangle$. Thus we may apply [5, Theorem 1] with the pairs of elements $(f_*(c_i), f_*(h))$ for $i = 1, \ldots, r$. Let $\tilde{A}$ be the finite covering corresponding to $(f|A)_* \circ \varphi : \pi_1(A) \longrightarrow K$. Using the Riemann-Hurwitz formula for the covering $\tilde{A} \longrightarrow A$, we get:

$$2\tilde{g} = 2 - \tilde{p} + \sigma \left(r - 1 - \sum_{i=1}^{i=r} \frac{1}{(\mu_i, \beta_i)} \right)$$

where $\tilde{g}$ denotes the genus of the orbit space of $\tilde{A}$, $\tilde{p}$ the number of boundary components, σ the degree of the (branched) covering induced on the orbit space of A by $\tilde{A} \longrightarrow A$ and β_i the order of the element $(f|A)_* \circ \varphi(c_i)$ in K. Since $(f|A)_* \circ \varphi(c_i) \notin \langle (f|A)_* \circ \varphi(h) \rangle$ then $(\mu_i, \beta_i) > 1$ and moreover it follows from Lemma 3.1 that $\tilde{p} = 1$. So we get:

$$2\tilde{g} \geq 1 + \sigma \left(\frac{r}{2} - 1 \right) \geq 1$$

which is impossible. This proves that the canonical tori of M do not degenerate.

4.2 Some results on finite coverings of Haken manifolds

Lemma 4.3 *Let A be a Seifert piece of M such that $f_*(\pi_1(A))$ is a non-abelian group. Let T be a component of ∂A. Then there exists a finite covering $\tilde{f} : \tilde{M} \longrightarrow \tilde{N}$ of f (that can be chosen regular) such that if $\tilde{A}$ is a lifting of A in $\tilde{M}$ then $\partial\tilde{A}$ contains at least two components T_1 and T_2 which project on T.*

Proof: Since $f_*(\pi_1(A))$ is a non-abelian group, we can find two elements g, h in $\pi_1(A)$ such that $k = [f_*(g), f_*(h)] \neq 1$. Since N is a Haken manifold, it follows from [6] that $\pi_1(N)$ is a residually finite group. Then there exists an epimorphism $\varphi : \pi_1(N) \longrightarrow K$, where K is a finite group such that $\varphi(k) \neq 1$. In particular, this implies that $K_A = \varphi(f_*(\pi_1(A)))$ is a non-abelian group which strictly contains $K_T = \varphi(f_*(\pi_1(T)))$. Let $\tilde{A}$ be the finite covering of A corresponding to $\varphi \circ (f|A)_*$. It follows from the construction above that the torus T lifts to n connected components in $\tilde{A}$, where $n = |K_A : K_T| > 1$ which completes the proof of the Lemma. ■

Lemma 4.4 *Let T be a separating canonical torus in M and let B be a component of $M - T$. Then there exists a finite covering $\tilde{f} : \tilde{M} \longrightarrow \tilde{N}$ of f (that can be chosen regular) such that if $\tilde{B}$ is a lifting of B in $\tilde{M}$ then $\partial\tilde{B}$ contains at least two connected components.*

Proof: If $f_*(\pi_1(B))$ is a non-abelian group we use the same construction as in the proof of Lemma 4.3. Then suppose that $f_*(\pi_1(B))$ is an abelian group (necessarily free since $\pi_1(N)$ is torsion free). Thus $(f|B)_* : \pi_1(B) \longrightarrow \pi_1(N)$ factors throught $H_1(B, \mathbf{Z})$. If the latter group is cyclic then so is $f_*(\pi_1(B))$ which is impossible since $T = \partial B$ is a non-degenerate torus. Thus since $H_1(B, \mathbf{Z})$ is non-cyclic the homomorphism induced by the inclusion $i_* : H_1(T, \mathbf{Z}) \longrightarrow H_1(B, \mathbf{Z})$ in not onto. Thus using the construction of [12, paragraphe 4.1.4], we can define a finite covering $\tilde{f} : \tilde{M} \longrightarrow \tilde{N}$ of f satisfying the conclusion of the Lemma. ■

Lemma 4.5 *Let T_1 and T_2 be two canonical tori in M such that the space $M - (T_1 \coprod T_2)$ is made of exactly two connected components. Let W be a connected component of $M - (T_1 \cup T_2)$. If $f_*(\pi_1(T_1)) \neq f_*(\pi_1(W))$ (resp. $f_*(\pi_1(T_2)) \neq f_*(\pi_1(W))$) there exists a finite covering $\tilde{f} : \tilde{M} \longrightarrow \tilde{N}$ of f (that can be chosen regular) such that if $\tilde{W}$ is a lifting of W in $\tilde{M}$ then $\partial\tilde{W}$ contains at least two connected components which project on T_1 (resp. on T_2).*

Proof: By symmetry we may assume that $f_*(\pi_1(T_1)) \neq f_*(\pi_1(W))$. Thus we can choose an element g in $\pi_1(W)$ such that $f_*(g) \notin f_*(\pi_1(T_1))$. Using [5, Theorem 1], we can find a finite group K and an epimorphism $\varphi : \pi_1(N) \longrightarrow K$ such that $\varphi(f_*(g)) \notin \varphi(f_*(\pi_1(T_1))) = K_{T_1}$. Let $\tilde{W}$ be the finite covering of W corresponding to $\varphi \circ (f|W)_*$. It follows from the above construction that the torus T_1 lifts to n connected components in $\tilde{W}$, where $n = |\varphi(f_*(\pi_1(W))) : K_{T_1}| > 1$ which completes the proof of the Lemma.

4.3 End of proof of Proposition 1.3

Lemma 4.6 *If there exists a finite covering $\tilde{f} : \tilde{M} \longrightarrow \tilde{N}$ of f which induces a homeomorphism from $(\mathcal{S}(\tilde{M}), \partial\mathcal{S}(\tilde{M}))$ to $(\mathcal{S}(\tilde{N}), \partial\mathcal{S}(\tilde{N}))$ then f satisfies the conclusion of Proposition 1.3.*

Proof: Suppose that such a covering exists. Denote by $\{A_1, \ldots, A_{s(M)}\}$ (resp. $\{B_1, \ldots, B_{s(N)}\}$) the Seifert pieces of M (resp. of N) corresponding to a minimal torus decomposition of M (resp. of N). Since $\tilde{f}$ is a homeomorphism between the Seifert pieces of M and those of N then for each $i \in \{1, \ldots, s(M)\}$ there exists an $\alpha_i \in \{1, \ldots, s(N)\}$ such that $f(A_i, \partial A_i) \subset (B_{\alpha_i}, \partial B_{\alpha_i})$. Moreover, since f is a degree one map, then the correspondance

$$\beta : \{1, \ldots, s(M)\} \in i \mapsto \alpha_i \in \{1, \ldots, s(N)\}$$

is onto. We next show that β is one to one. For this suppose that A_1 and A_2 are sent to the same B_α by f. Fix a base point $*$ in B_α and choose a base point x_i in A_i such that $f(x_i) = *$, for $i = 1, 2$. Denote by $(\tilde{B}_\alpha, \tilde{*})$ the induced covering over $(B_\alpha, *)$ by $\tilde{N}$. Then this covering induces via $f_i = f|A_i$ a covering $(\tilde{A}_i, \tilde{x}_i)$ over (A_i, x_i) and we get $f(\tilde{A}_i) \subset \tilde{B}_\alpha$ for $i = 1, 2$ which is impossible. Thus $f_i = f|(A_i, \partial A_i) : (A_i, \partial A_i) \longrightarrow (B_{\alpha_i}, \partial B_{\alpha_i})$ are degree one maps. Moreover, since β is bijective and since f is a **Z**-homology equivalence, $f_i|\partial A_i : \partial A_i \longrightarrow \partial B_{\alpha_i}$ is a homeomorphism. It remains to show that each f_i can be homotoped (rel ∂A_i) to a homeomorphism. Let $\tilde{B}_{\alpha_i}$ (resp. $\tilde{A}_i$) be a component of the covering induced by $\tilde{N} \longrightarrow N$ (resp. by $\tilde{N} \longrightarrow N$ via f_i) over B_{α_i} (resp. A_i). Then consider the following commutative diagram:

$$\begin{array}{ccc}
(\tilde{A}_i, \partial\tilde{A}_i) & \xrightarrow{\tilde{f}_i} & (\tilde{B}_{\alpha_i}, \partial\tilde{B}_{\alpha_i}) \\
\downarrow p_i & & \downarrow q_i \\
(A_i, \partial A_i) & \xrightarrow{f_i} & (B_{\alpha_i}, \partial B_{\alpha_i})
\end{array}$$

where $\tilde{f}_i$ is a homeomorphism. Since $\deg(f_i) = \deg(\tilde{f}_i) = 1$ then the degrees of the coverings p_i and q_i are equal and thus f induces isomorphisms between $\pi_1(A_i)$ and $\pi_1(B_{\alpha_i})$ which ends the proof.

■

This result means that in the following of the proof of Proposition 1.3, we are at liberty to replace the given map $f : M \longrightarrow N$ by any finite covering $\tilde{f} : \tilde{M} \longrightarrow \tilde{N}$ of f. Moreover, since the canonical tori of M do not degenerate, we may apply [3, Lemma 3.1] to $f : M \longrightarrow N$ that implies that we can always assume (up to finite covering on f) that the components of $\mathcal{S}(M)$ are either of type I, which corresponds to Seifert fibered spaces whose orbit space is a surface of genus ≥ 3 or of type II, which corresponds to Seifert fibered spaces whose orbit space is an annulus and whose π_1 is sent onto a rank two abelian free group via f_*.

In the following, we call *mixed tori* the canonical tori of M (resp. of N) which are a common boundary component of a hyperbolic piece and of a Seifert fibered piece. Let $\mathcal{T}'(M)$ (resp. $\mathcal{T}'(N)$) be the set of mixed tori ($\partial S_{(.)} = W'_{(.)}$ and $\partial H_{(.)} = W'_{(.)}$). Thus we get the following result:

Lemma 4.7 *There exists a homotopy $(f_t)_{0\leq t\leq 1}$ on f such that $f_0 = f$ and satisfying the following properties: f_1 is transversal to $\mathcal{T}'(N)$ and f_1 induces two maps $f_1^- = f_1|(S_M, \partial S_M) : (S_M, \partial S_M) \longrightarrow (S_N, \partial S_N)$ and $f_1^+ = f_1|(H_M, \partial H_M) : (H_M, \partial H_M) \longrightarrow (H_N, \partial H_N)$ such that $f_1^-|\partial S_M : \partial S_M \longrightarrow \partial S_N$ (and obviously $f_1^+|\partial H_M : \partial H_M \longrightarrow \partial H_N$) are homeomorphisms.*

Proof: We know that $f(\mathcal{H}(M)) \subset \overset{\circ}{H}_N$. It follows from Lemma 4.1, that for each component T of $\mathcal{T}(M)$, $f|T$ is a non-degenerate map and in particular $f|\mathcal{S}(M)$ is a non-degenerate map in the sense of Jaco and Shalen. Applying the Jaco-Shalen *Mapping Theorem* with some care we may assume that $f(\mathcal{S}(M)) \subset \mathrm{int}(\mathcal{S}(N))$ without change $f|\mathcal{H}(M)$. Then, after modifying f by a homotopy supported on a regular neighbourhood $W(T)$ of T, we show that for each $T \in \mathcal{T}'(M)$, there exists a torus U in $\mathcal{T}'(N)$ such that $f(T) \subset U$. Let A and B be two geometrical pieces of M which are adjacent to T. Since $T \in \mathcal{T}'(M)$, we may assume that $A \in \mathcal{S}(M)$ and $B \in \mathcal{H}(M)$ and thus $f(A') \subset \mathrm{int}(\mathcal{S}(M))$ and $f(B') \subset \mathrm{int}(\mathcal{H}(N))$, where $A' = A - W(\mathcal{T}(M))$ and $B' = B - W(\mathcal{T}(M))$, where $W(\mathcal{T}(M))$ denotes a regular neighbourhood of $\mathcal{T}(M)$. Thus $f(W(T))$ contains necessarily a component of $\partial\mathcal{S}(N) \cap \partial\mathcal{H}(N)$. Thus, modifying f by a homotopy, fixing $f|M - W(T)$, we can find a component U of $\partial\mathcal{S}(N) \cap \partial\mathcal{H}(N)$ (thus of

$\mathcal{T}'(N)$) such that $f(T) \subset U$. We proceed as above for all components of $\mathcal{T}'(M)$.

Let T be a mixed torus in N. Using standard cut and paste arguments and the fact that $\partial\mathcal{S}(M)$ and $\partial\mathcal{H}(M)$ are incompressible, we may change f by a homotopy fixing $f|\mathcal{H}(M) \cup \mathcal{S}(M)$ such that $f^{-1}(T)$ is a collection of incompressible surfaces. Since $f^{-1}(T) \subset M \setminus (\mathcal{H}(M) \cup \mathcal{S}(M))$ it is an union of parallel copies of incompressible tori in $\mathcal{T}(M) \times I$. So we can show, after modifying f by a homotopy, that for each component $T \in \mathcal{T}'(M)$, the set $f^{-1}(\mathcal{T}'(N)) \cap W(T)$ is made of a single component.

This allows to define the promised maps f_1^+ and f_1^- of the Lemma. Set $\mathcal{T}'(M) = \{T_1, \ldots, T_m\}$ and $\mathcal{T}'(N) = \{U_1, \ldots, U_n\}$. The above paragraph allows to define a surjective correspondance $\beta : \{1, \ldots, m\} \ni i \mapsto \alpha_i \in \{1, \ldots, n\}$ given by $f(T_i) \subset U_{\alpha_i}$. To complete the proof of the Lemma it is sufficient to state the following Lemma (notations are the same as above):

Lemma 4.8 *For each $i \in \{1, \ldots, m\}$, the map $f_i = f|T_i : T_i \longrightarrow U_{\alpha_i}$ is homotopic to a homeomorphism and the map β is one to one (and so bijective).*

Proof: We first prove that for each $i \in \{1, \ldots, m\}$, the map $f_i = f|T_i : T_i \longrightarrow U_{\alpha_i}$ is homotopic to a homeomorphism. Suppose first that T_i is a non-separating torus in M. So we can find a simple closed curve γ in M such that $[T_i].[\gamma] = 1$. Since f is a $\mathbf{Z}$-homology equivalence it preserves the intersection number and thus

$$[T_i].[\gamma] = \deg(f_i) \times [U_{\alpha_i}].[f_*(\gamma)] = 1$$

In particular $\deg(f_i) = 1$ and thus f_i is homotopic to a homeomorphism. Suppose that T_i is a separating torus and let A and B be two components of $M - T_i$. If $H_1(A, \mathbf{Z})$ and $H_1(B, \mathbf{Z})$ are not isomorphic to $\mathbf{Z}$ then the homomorphisms $H_1(T_i, \mathbf{Z}) \longrightarrow H_1(A, \mathbf{Z})$ and $H_1(T_i, \mathbf{Z}) \longrightarrow H_1(B, \mathbf{Z})$ are not surjective. Combining this remark with the fact that f is a $\mathbf{Z}$-homology equivalence we can find a finite covering of f in which T_i lifts to non-separating tori. So suppose that $H_1(A, \mathbf{Z}) = \mathbf{Z}$ (say). Then $f_*(\pi_1(A))$ is non-abelian (otherwise $(f|A)_*$ factors throught $H_1(A, \mathbf{Z}) = \mathbf{Z}$ and then $f_*(\pi_1(A))$ is a rank one abelian free group which is impossible). Since $\pi_1(N)$ is a residually finite group, we can find a finite covering $\tilde{A}$ of A, induced from some finite covering of N by $f|A$, such that ∂A is disconnected. We use the same argument for B which allows to suppose that T_i is a non-separating torus in some finite covering of f.

Remark 1 *The above paragraph implies the following: If a Seifert piece A of M has at least one boundary component in $\mathcal{T}'(M)$ then A is necessarily* *of type I. To see this, it is sufficient to use arguments similar to those of [3, Lemma 3.6].*

To complete the proof of the Lemma it remains to show that if $i \neq j$ then $U_{\alpha_i} \neq U_{\alpha_j}$. Suppose the contrary and denote by U the torus of $\mathcal{T}'(N)$ such that $U = U_{\alpha_i} = U_{\alpha_j}$. Denote by A_i and A_j the components of $\mathcal{S}(M)$ adjacent to T_i and T_j (necessarily of type I by the above remark).

Claim 3 *With the hypothesis and notations above we claim that necessarily $A_i = A_j = A$.*

Proof of Claim : Let B_i and B_j be the components of $\mathcal{S}(N)$ such that $f(A_i) \subset B_i$ and $f(A_j) \subset B_j$. Since $U \in \mathcal{T}'(N)$ and since B_i and B_j contain U then necessarily $B_i = B_j$. Moreover, since A_i and A_j are of type I, then by [12, Lemme 3.2 and 4.3.4], we get $A_i = A_j = A$. ■

In the following we denote by B the Seifert fibered piece of N such that $f(A) \subset B$. Let H_i and H_j be the hyperbolic pieces of M adjacent to T_i and T_j. Then we consider the manifold $X = M - (T_i \cup T_j)$. We distinguish the following casis:

First Case: Assume first that X has three connected components. Denote by C that which contains A, by B_i that which contains H_i and by B_j that which contains H_j. Since A is of type I then $f_*(\pi_1(A))$ is a non-abelian group and thus by Lemma 4.3, we can find a finite covering $\tilde{M}$ of M, induced from some finite covering $\tilde{N}$ of N such that if $\tilde{A}$ is a component over A (in $\tilde{C}$) then $\partial\tilde{A}$ has at least two boundary components T_i^1 and T_i^2 which project to T_i and T_j^1 and T_j^2 which project to T_j. Moreover, applying Lemma 4.4 to B_i and B_j we may assume, by taking fiber products of coverings, that the components $\tilde{B}_i$ and $\tilde{B}_j$, over B_i and B_j, have at least two boundary components. In $\tilde{N}$, denote by U_1 the component such that $f(T_i^1) = U_1$, $f(T_j^1) = U_1$ and by U_2 the component such that $f(T_i^2) = U_2$, $f(T_j^2) = U_2$. So we may choose a simple closed curve γ_i (resp. γ_j) in $\tilde{A} \cup \tilde{B}_i$ (resp. $\tilde{A} \cup \tilde{B}_j$) such that γ_i (resp. γ_j) cuts transversally in a single point T_i^1 and T_i^2 (resp. T_j^1 et T_j^2) (see figure 3). Since f is a $\mathbf{Z}$-homology equivalence it must preserve the intersection number and thus we have $[U_1].[f_*(\gamma_i)] = \pm 1$ and $[U_1].[f_*(\gamma_j)] = \pm 1$. Now we define the curve $\gamma = \gamma_i \sharp \gamma_j$. By contruction we know $[T_i^1].[\gamma] = \pm 1$ and it follows from the above argument that $[U_1].[f_*(\gamma)] = 0$ or ± 2 which gives a contradiction since f is a $\mathbf{Z}$-homology equivalence. This proves that the first case is impossible.

Second Case: Assume that X is made of two components denoted by V and W. We examin two situations:

Figure 3: X has three connected components

First Subcase: First suppose that V contains A and H_j and W contains H_i. We can define, using lemmas 4.3 and 4.4, a finite covering of M similar to that of the first case with components V and W (we double the components over T_i in $\tilde{A}$ and in $\tilde{W}$, see figure 4). We use the same notations as in the first case. Let $\tilde{A}$ be a component over A, $\tilde{H}_i$ a component over H_i connected to $\tilde{A}$ and $\tilde{W}$ a component over W connected to $\tilde{A}$. We choose two simple closed curves c_W and c_V in the following way: c_W is a simple closed curve in $\tilde{A} \cup \tilde{W}$ cutting two tori of $\partial \tilde{A} \cap \partial \tilde{W}$ in a single point and transversally and c_V is a simple closed curve in $\tilde{A} \cup \tilde{V}$ cutting at least one component over T_i in a single point and transversally. We now argue as in the first case with the curve c defined by the connected sum $c = c_V \sharp c_W$ to get a contradiction.

Second Subcase: Assume now that V contains A and that W contains H_i and H_j. If $f_*(\pi_1(T_i)) = f_*(\pi_1(T_j)) = f_*(\pi_1(W))$ then $f_*(\pi_1(W))$ is a rank 2 free abelian group. Let H be the geometric piece adjacent to B along U.

Figure 4: X has two connected components

In this case, W is made of hyperbolic manifolds and we may assume, up to homotopy, that $f(W') \subset \text{int}(H)$, where $W' = W - (W(T_i) \cup W(T_j))$. On the other hand, we may find a simple closed curve γ in $A \cup W$ which cuts T_i and T_j transversally in a single point. The above construction shows that $[U].[f_*(\gamma)] = 0$ or 2 which gives a contradiction. Suppose now that $f_*(\pi_1(T_i)) \neq f_*(\pi_1(W))$. It follows from Lemma 4.5 that we may find a finite covering $\tilde{M}$ of M induced by f from some finite covering $\tilde{N}$ of N such that if $\tilde{A}$ denotes a component over A and $\tilde{W}$ denotes a component over W connected to $\tilde{A}$ then T_i lifts to a space which contains at least two connected component T_i^1 and T_i^2 in $\tilde{A}$ and $\tilde{W}$. Let T_j^1 be a component over T_j and U_1 be a component over U (in $\tilde{N}$) such that $\tilde{f}(T_i^1) \subset U_1$ and $\tilde{f}(T_j^1) \subset U_1$. So we may choose two disjoint simple closed curves c and d in $\tilde{M}$ in the following way: c is in $\tilde{A} \cup \tilde{W}$, cuts T_i^1 and T_i^2 in a single point and d is in

$\tilde{A} \cup \tilde{W}$, cuts T_i^2 and T_j^1 in a single point. We proceed as in the above casis with the simple closed curve $\gamma = c \sharp d$ which gives a contradiction.

Figure 5: X has two connected components

Third Case: Finally assume that X is connected. We may choose a curve c_i which cuts T_i transversally in a single point and a curve c_j which cuts T_j transversally in a single point such that $c_i \cap c_j = \emptyset$. We proceed as in the above casis with the curve $\gamma = c_i \sharp c_j$ which gives a contradiction.

■

To complete the proof of Proposition 1.3, it remains to show that $f_1^- : (S_M, \partial S_M) \longrightarrow (S_N, \partial S_N)$ is homotopic to a homeomorphism (rel ∂S_M). Since by Lemma 4.7, the map $f_1^- | \partial S_M : \partial S_M \longrightarrow \partial S_N$ is a homeomorphism, we now can use the results of [3, Section 3] to prove that $(S_M, \partial S_M) \overset{f_1^-}{\simeq} (S_N, \partial S_N)$.

5 Proof of Theorem 1.2

In this section, we deal with a map $f : M \longrightarrow N$ between closed Haken manifolds sucht that each finite covering of f is a **Z**-homology equivalence

Figure 6: X is connected

and satisfying $f(\mathcal{H}(M)^+) \subset H_N$. If the Jaco Shalen Johannson decomposition of M is trivial (i.e. $\mathcal{T}(M) = \emptyset$) this means that M is geometric (i.e. M is either a Seifert fibered manifold with infinite fundamental group or a hyperbolic manifold). If M is a Seifert fibered manifold then since $0 = \|M\| \geq \|N\| = 0$, N is a graph manifold and then Theorem 1.2 is a special case of [12, Proposition 0.1]. If M is a hyperbolic manifold, then since $\pi_1(N) = f_*(\pi_1(M))$, M is necessarily non-degenerate and thus $N - \mathcal{T}(N)$ is made of hyperbolic pieces and Theorem 1.2 is clearly true ($S_M = S_N = \emptyset$). Hence, in the following we may assume that $\mathcal{T}(M) \neq \emptyset$. If each canonical torus of M is non-degenerate then we can apply Proposition 1.3. So we assume that some canonical tori of M degenerate under f.

5.1 Maximal Ends

In this paragraph, we construct, using the degenerate tori of M, a finite collection of codimension zero submanifolds of M which contain all the degeneration of the map f. First of all we recall the following definition which can be found in [3]:

Definition 5.1 *A codimension zero submanifold A of M is said to be a maximal end of M if it satisfies the following properties:*

(i) *∂A is made of one component of $\mathcal{T}(M)$ and $f_*(\pi_1(A)) = \mathbf{Z}$,*

(ii) *for each finite covering $p : \tilde{M} \longrightarrow M$ of M, induced by f from some finite covering of N, the components of $p^{-1}(A)$ satisfy point* (i),

(iii) *if B is a submanifold of M containing A and satisfying points* (i) *and* (ii) *then $B = A$.*

Fix a degenerate torus T in M. Let A be the component of $M \setminus T$ which satisfies the conclusion of Lemma 3.1 and set $B = M - A$. Denote by Q the component of B adjacent to A along T. We first state the following:

Claim 4 *If Q is not degenerate then A is a maximal end of M and if Q degenerates then there exists an end of M, denoted by E, such that $E \supset A \cup Q$.*

Proof of Claim: Assume that Q does not degenerate. Then A is maximal otherwise it would exist an end E of M containing $Q \cup A$ and then $f_*(\pi_1(Q)) \subset f_*(\pi_1(E))$ would be a cyclic group which is impossible. It follows from Lemma 4.2 that $f_*(\pi_1(A)) \neq \{1\}$ (and thus $f_*(\pi_1(A)) = \mathbf{Z}$ since A is an end of M). It remains to prove that each finite covering $\widetilde{A}$ of A which comes (via $f|A$) from some finite covering of N, satisfies point (ii) of definition 5.1. To see this, we can use arguments similar to those of the proof of [3, Lemma 4.1]. Suppose now that Q degenerates. If Q is a hyperbolic manifold then $f_*(\pi_1(Q)) = \mathbf{Z}$. In particular this means that each component of ∂Q degenerates under f. Denote by $T_1, \ldots, T_k$ the components of $\partial Q - T$ and by $A_1, \ldots, A_k$ the components of de $B \setminus Q$ such that for each i we have $\partial A_i = T_i$. We first claim that there exists exactly one A_i which is not an end of M. Indeed suppose that each A_i is an end (i.e. each A_i satisfies the conclusion Lemma 3.1). We show that this case is impossible. Indeed, using the Van-Kampen's Theorem for the decomposition

$$M = A \bigcup_T Q \bigcup_{T_1} A_1 \bigcup_{T_2} \ldots \bigcup_{T_k} A_k$$

of M we get: $\pi_1(N) = f_*(\pi_1(Q))$. Since Q is a degenerate hyperbolic manifold then $\pi_1(N)$ is a cyclic group and therefore N is a solid torus, which is impossible. Suppose now that there exist two components, says A_i and A_j, which are not ends of M. Then we get a contradiction using Lemma 3.1 (since otherwise we could include A_j in an end of M which is absurd). Denote by i the integer of $\{1, \ldots, k\}$ such that A_i is not an end of M. Then by Lemma 3.1, the submanifold

$$A' = A \bigcup_T Q \bigcup_{j \neq i} A_i$$

is an end of M. Then, here we may "extend" the submanifold A. This proves Claim 4 when Q is a hyperbolic manifold.

Assume now that Q is a Seifert fibered manifold. Since $f|Q$ is a degenerate map, in the sense of Jaco and Shalen, this means that we have to consider the two following casis:

(i) $f_*(\pi_1(Q))$ is a cyclic group or,

(ii) $f_*(h) = 1$ in $\pi_1(N)$, where h denotes the homotopy class of the regular fiber of Q.

In case (i) we can, as for the hyperbolic degenerate case, extend A in such a way that it contains Q. Then assume that $f_*(h) = 1$. Since h is represented in each component of ∂Q by a primitive curve then each component of ∂Q degenerates. Then we use the same notations as in the hyperbolic case. If there is an A_i which is not an end of M we can "extend" A. So assume that each A_i satisfies the conclusion of Lemma 3.1. We show that this case is impossible. Since $f_*(h) = 1$ and since $\pi_1(N)$ is torsion free then $f_*(\gamma) = 1$ for all fibers γ of Q. So the map $(f|Q)_* : \pi_1(Q) \longrightarrow \pi_1(N)$ factors throught $\pi_1(Q)/\langle \text{all fibers} \rangle$ which is the fundamental group of an orientable surface V. Moreover since $f_*(\pi_1(A))$ and $f_*(\pi_1(A_i))$ are cyclic groups for $i = 1, \ldots, k$ and since $H_1(A, \mathbf{Z}) = H_1(A_i, \mathbf{Z}) = \mathbf{Z}$ then $f_*(\pi_1(\partial A)) = f_*(\pi_1(A))$ and $f_*(\pi_1(\partial A_i)) = f_*(\pi_1(A_i))$ for $i = 1, \ldots, k$. Thus applying the Van-Kampen Theorem to the decomposition

$$M = A \bigcup Q \bigcup_{i=1,\ldots,k} A_i$$

we know that the map $f_* : \pi_1(M) \longrightarrow \pi_1(N)$ factors throught $\pi_1(V)$. Let D_Q be a $K(\pi_1(V), 1)$ space. Note that it follows from the construction that $H_3(D_Q, \mathbf{Z}) = 0$. Then it follows from the asphericity of M, V and N that there exists maps α, β such that the following diagram commutes:

$$\begin{array}{ccc} M & \xrightarrow{f} & N \\ {\scriptstyle\alpha}\downarrow & \nearrow{\scriptstyle\beta} & \\ D_Q & & \end{array}$$

Since $H_3(D_Q, \mathbf{Z}) = 0$ this contradicts the fact that $f_* : H_3(M; \mathbf{Z}) \longrightarrow H_3(N; \mathbf{Z})$ is an isomorphism.

■

The proof of Claim 4 implies that if T is a degenerate torus of M and if A is an end of M with $\partial A = T$ then either A is a maximal end (when A is adjacent along T to a non-degenerate geometrical piece) or there exists an end $E \supseteq A$ in M such that $E \neq A$. We set $E_0 = A$ and $E_1 = B$. If the geometrical piece adajacent to E_1 along ∂E_1 is degenerate, we carry on this process which allows to define a strictly increasing sequence of ends $E_0 \subseteq E_1 \subseteq \ldots \subseteq E_n$ in M. If this process does not stop (i.e. if each geometrical piece of M degenerates) we may assume that E_n is adjacent along $T_n = \partial E_n$ to a geometrical piece C_n such that $M = E_n \cup_{T_n} C_n$. If C_n is hyperbolic then $\pi_1(N)$ is a cyclic group and if C_n is a Seifert piece then either $\pi_1(N)$ is a cyclic group or, by the same arguments as above we can show that f factors throught a map $\beta : D \longrightarrow N$, where $H_3(D, \mathbf{Z}) = 0$ which is impossible (since f is a degree one map). So when $\mathcal{T}(M) \neq \emptyset$, M always contains some non-degenerate geometrical pieces which implies that the above process does stop. So the above arguments clearly imply the following result:

Lemma 5.2 *If $\mathcal{T}(M) \neq \emptyset$, there exists a finite collection $\mathcal{A}$ of maximal ends in M such that each geometrical component of $M \setminus \mathcal{A}$ does not degenerate under f.*

5.2 Factorization of the map f

Let A be a maximal end of M and set $T = \partial A$. We denote by C the geometrical component of $M - \mathcal{T}(M)$ adjacent to A along T. Necessarily C is non-degenerate, since A is maximal. Then we consider two casis depending on the fact that C is a Seifert fibered space or a hyperbolic manifold.

Lemma 5.3 *If C is a Seifert fibered manifold then there exists a Haken manifold M_1 obtained from M by a collpase π along A and a map $f_1 : M_1 \longrightarrow N$ such that $f = f_1 \circ \pi$. Moreover*

$$\pi(A \cup_T C, \partial(A \cup_T C)) = (C_1, \partial C_1)$$

where C_1 is a (non-degenerate) Seifert piece of M_1 obtained by Dehn filling on C along T and all finite coverings of f_1 are $\mathbf{Z}$-homology equivalences.

Proof: Let λ be the primitive curve of T such that $f_*(\lambda) = 1$ in $\pi_1(N)$ and let V be a solid torus whose meridian (resp. core) is denoted by m (resp. l). Let M_1 be the manifold obtained from $\overline{M - A}$ by identifying the meridian of V to λ. Let C_1 be the manifold $C \cup_T V$. Since C is non-degenerate, then $f_*(h) \neq 1$, where h denotes the homotopy class of the regular fiber of C, and thus the fibration of C extends to a Seifert fibration of C_1. On the other hand, using similar arguments to those of [3, paragraph 5.1.2], we may show that there exists a map $f_1 : M_1 \longrightarrow N$

satisfying hypothesis of Theorem 1.2 such that f factors throught M_1, via the collapsing projection

$$\pi : M = (\overline{M - A}) \cup A \longrightarrow (\overline{M - A}) \cup V = M_1$$

This completes the proof of the Lemma.

■

Lemma 5.4 *If C is a hyperbolic manifold then there exists a hyperbolic manifold M_1 obtained from M by a collapse π along A and a map $f_1 : M_1 \longrightarrow N$ such that $f = f_1 \circ \pi$. Moreover*

$$\pi(A \cup_T C, \partial(A \cup_T C)) = (C_1, \partial C_1)$$

where C_1 is a union of hyperbolic pieces of M_1 obtained by Dehn filling on C along T and all finite coverings of f_1 are $\mathbf{Z}$-homology equivalences.

Proof: We use the same process as in the proof of the above Lemma, by performing Dehn fillings on C along T with the same notations. But since C is a hyperbolic manifold, the Haken manifold C_1 is not geometric in general and can be complicated. Here the main idea is to show that we do not “introduce" some Seifert pieces in C_1. More precisely we show that we may assume that all pieces of $C_1 - W_{C_1}$ are hyperbolic. Let $\pi_1 : (C_0 = A \cup C, \partial C_0 = \partial(A \cup C)) \longrightarrow (C_1, \partial C_1)$ be the map induced by the collpasing projection and which is equal to the identity in the boundary. Since C_1 is a Haken manifold with incompressible toral boundary there exists a finite collection of incompressible tori in $\text{int}(C_1)$ which cut C_1 to Seifert fibered manifold and complete finite volume hyperbolic manifolds.

Claim 5 *The (possible) Seifert pieces of $C_1 - W_{C_1}$ do degenerate under f_1.*

Proof of Claim : Let $\mathcal{H}_1$ be the connected component of $\mathcal{H}(N)$ such that $f(C) \subset \mathcal{H}_1$. Then $f_1(C_1) \subset \mathcal{H}_1$. If C_1 contains a Seifert fibered piece which does not degenerate under f then it follows from [7, Mapping Theorem] that there exists a Seifert piece in $\mathcal{H}_1$ which is impossible since $\mathcal{H}_1$ is only made of hyperbolic manifolds.

■

Now, since every Seifert fibered pieces of C_1 are degenerate, some canonical tori of W_{C_1} do degenerate under f_1. So we can start again as in the first step by defining the maximal ends of C_1. Using this process we get a sequence of Haken manifolds $C_1, \ldots, C_n$, collapsing projections $\pi_i : C_{i-1} \longrightarrow C_i$, and maps $f_i : C_i \longrightarrow N$ such that $f = f_i \circ \pi_i$. Note that if C_i contains some degenerate canonical tori (in particular if it contains some Seifert fibered spaces), then $\|C_i\| > \|C_{i+1}\|$ which garanties that the C_i's are never homeomorphic. We now show that there exists an integer n_0 such that C_{n_0} contains no degenerate tori (in particular it contains no Seifert fibered pieces). Indeed, suppose the contrary. Then the above process would allow us to define an infinite sequence of degree one maps:

$$(C_0, \partial C_0) \xrightarrow{\pi_1} (C_1, \partial C_1) \xrightarrow{\pi_2} \cdots$$

$$\xrightarrow{\pi_n} (C_n, \partial C_n) \xrightarrow{\pi_{n+1}} (C_{n+1}, \partial C_{n+1}) \longrightarrow \cdots$$

Since the C_i's are pairwise non-homeomorphic, no map π_i can be homotoped to a homeomorphism which contradicts [14, Theorem 1]. This completes the proof of the Lemma.

5.3 End of proof of Theorem 1.2

Let $\mathcal{A} = \{A_1, \ldots, A_n\}$ be the maximal ends of M. For each A_i, we apply Lemma 5.3 or Lemma 5.4. This allows us to define a Haken manifold M_1 and a map $f_1 : M_1 \longrightarrow N$ throught which $f : M \longrightarrow N$ factors. Denote by $\pi : M \longrightarrow M_1$ the collapsing projection such that $f = f_1 \circ \pi$. By Lemmas 5.3 and 5.4, the Seifert pieces of M_1 are obtained by (Seifert) Dehn fillings along those of M, f_1 satisfies the homological hypothesis of Proposition 1.3 and the hyperbolic pieces of M_1 do not degenerate. So to complete the proof of Theorem 1.2 it is sufficient to apply Proposition 1.3 to $f_1 : M_1 \longrightarrow N$.

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