THE CRM APPLIED MATHEMATICAL AND PHYSICS (CAMP) SEMINARS

CENTRE DE RECERCA MATEMÀTICA

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Numerical computation of stability boundaries for structured population models.

Abstract: Population models describe the changes in the number of individuals of a population due to death and reproduction. In nature we observe that some physiological characteristics, like the age or the body size of the individual, play an important role on its behavior. Physiologically structured population models express the dynamics of the population in terms of the processes taking place at the individual level, considering physiological differences [6].

We consider a model for an age-size structured consumer population feeding by an unstructured resource [3]. The model has a trivial and a positive equilibrium [2]. Taking into account that the model has some parameters, letting two parameters free and fixing the others, it is possible to express the existence and stability properties of a positive equilibrium in terms of those parameters. We define the existence boundary in a parameter plane, as the curve that separates the plane into the region of parameters such that a positive equilibrium exists and the region where it does not exist. In the same way we define the stability boundary as the curve that separates the existing region of the plane into the region where the positive equilibrium is stable and the region where it is unstable [5].

We present a graphical user interface (GUI) that computes existence and stability boundaries for structured population models of the type of [3]. The interface implements algorithms proposed in [2] with new ones. With this interface, the user can compute stability boundaries in parameters plane by entering directly the biological ingredients of the model. The user does not need to derive the existence and stability conditions for a positive equilibrium from the population equation, this is already implemented from the results of [2] into the code. This interface combines numerical continuation [1], Runge-Kutta methods [4], and numerical integration. We had also implemented a new mechanism to look for an initial value to start the continuation.

References

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