ADVANCED COURSE “COMPACTIFYING MODULI SPACES”

Titles and abstracts

Valery Alexeev

Moduli of weighted stable hyperplane arrangements, with applications

Contents:

A stable pair(X, B) consists of a projective variety X and a $\mathbb{Q}$-divisor $B = \sum_{i=1}^{n} b_i B_i$ on it, so that the pair has reasonable (slc) singularities and the divisor $K_X + B$ is ample. It is a higher-dimensional generalization of a weighted stable n-pointed curve.

While various existence theorems about the moduli spaces of stable pairs have been proved, relatively few situations are known where the pairs and their moduli spaces can be explicitly computed. (Compare this with the situation for curves, where for any fixed genus g and weight (b_i) both the stable curves and their moduli space$\overline{M}_{g,(b_i)}$ are fairly easy to understand).

One large class where explicit computations are possible is the class of weighted stable hyperplane arrangements. They provide compactifications for the moduli spaces of log canonical hyperplane arrangements $(\mathbb{P}^k, \sum_{i=1}^{n} b_i B_i)$. As applications, one also obtains various results about moduli spaces of surfaces of general type and K3 surfaces, typically by considering Galois covers of $\mathbb{P}^2$ ramified in special configurations of lines.

In these talks, I will try to explain, as concretely as possible, how to work with such weighted stable pairs, and how to make computations about them and their moduli spaces. The whole story is an intricate interplay of Minimal Model Program, Geometric Invariant Theory, Matroid theory, and polytopal tilings. It is quite combinatorial in spirit and in methods.

Paul Hacking

Compact moduli spaces of surfaces and exceptional vector bundles

Contents:

The moduli space of surfaces of general type has a natural compactification due to Kollár and Shepherd-Barron which is analogous to the Deligne-Mumford compactification of the moduli space of curves. However, very little is known about this moduli space or its compactification in general (for example it can have many irreducible components and be highly singular). A key question is to enumerate the boundary divisors in cases where the moduli space is well behaved. The most basic boundary divisors are those given by degenerations of the smooth surface to a surface with a cyclic quotient singularity of a special type, first studied by J. Wahl. We will describe a construction which relates these boundary divisors to the classification of stable vector bundles on the smooth surface in the case $H^{2,0} = H^1 = 0$. In particular we will connect with the theory of exceptional collections of vector bundles used in the study of the derived category of coherent sheaves.

My lectures will review the necessary background material and there will be a strong emphasis on examples. In particular we will discuss the examples of del Pezzo surfaces and surfaces of general type with $K^2 = 1$ (based on work by my graduate student Anna Kazanova).

Radu Laza

Perspectives on the compactification problem for moduli spaces

Contents:

In these lectures, we will discuss various approaches to constructing and compactifying moduli spaces. We will discuss various pros and cons for each approach, as well as some comparison results between these approaches. The overarching theme of the lectures is that each approach sheds light on a different aspect of the moduli problem under consideration, but taken together these approaches give a fuller picture of the moduli space and its compactification. Specifically, we will cover

1. GIT and VGIT - a classical approach, which rarely gives a modular compactification;
2. Hodge Theory - an approach that gives a lot of structure to the moduli space, but it is rarely applicable;
3. KSBA approach (based on the minimal model program) - an approach that gives a modular compactification for a large class of examples, but which is difficult to apply in practice;
4. Examples and Open Questions (curves, del Pezzo surfaces, cubic hypersurfaces, K3 surfaces, etc.).

Manfred Lehn

Moduli of rational curves

Contents:

Rational normal curves. Grassmannians of lines. Conics. Complete conics. Hilbert schemes of curves. Chow varieties. Moduli of stable maps. (Differences between these constructions.) Tangent constructions, obstructions, deformation theory, singularities. Twisted cubics and moduli. Matrix representations and invariant theoretic constructions. The Beauville-Donagi example. The space of twisted cubics on a cubic fourfold.

Dragos Oprea

The moduli space of stable quotients

Contents:

I will introduce and describe the moduli space of stable quotients which provides a compactification of the space of maps from curves to Grassmannians. These moduli spaces admit virtual fundamental classes allowing for the definition of enumerative invariants. I will also describe calculations and applications related to stable quotients.

Note: each course consists of 4 lectures of 1 hour each.