Title: (Non)existence of integrals that are polynomial in momenta

Abstract. I will consider natural Hamiltonian systems with two degrees of freedom and speak about the existence and nonexistence of integrals that are polynomial in momenta. This is a classical topic, I will give a small historical overview and explain the classical and modern motivation. In the mathematical part of my talk, I will mostly discuss the following question: given a metric, how to prove the (non)existence of an integral of a given degree, and how to find it explicitly? I will show one trick and one method to do it. As an application of the trick, I will present a new metric on the sphere whose geodesic flow admits an integral of degree 3 in momenta, this result is joint with Shevchishin. As an application of the method, I give a solution of a problem explicitly stated by J.Brink; this result is joint with Kruglikov.