Instability of high dimensional Hamiltonian Systems: Multiple resonances do not impede diffusion.

Amadeu Delshams, Rafael de la Llave and Tere M. Seara

Abstract. We consider models given by Hamiltonians of the form

$$H(I,\varphi,p,q,t;\varepsilon) = h(I) + \sum_{j=1}^{n} \pm \left(\frac{1}{2}p_j^2 + V_j(q_j)\right) + \varepsilon Q(I,\varphi,p,q,t;\varepsilon)$$

where I,φ are d-dimensional actions and angles, p,q are n-dimensional real conjugated variables, and t is an angle.

These are higher dimensional analogues, both in the center and hyperbolic directions, of the models studied in previous papers by the authors. All these models present the large gap problem. We show that, for ε small enough, under regularity and explicit non-degeneracy conditions on the model, there are orbits whose action variables I perform rather arbitrary excursions in a domain of size $O(1)$. This domain includes resonance lines and, hence, large gaps among d-dimensional KAM tori. The main new phenomenon that appears when the dimension d of the center directions is larger than one, is the existence of multiple resonances. We show that, since these multiple resonances happen in sets of codimension greater than one in the space of actions I, they can be contoured. This corresponds to the mechanism called diffusion across resonances in the Physics literature.