

Advanced Course on Geometry and Dynamics of Integrable Systems



Objectives

This Advanced course aims at describing different aspects in the study of integrable systems from a geometrical, algebraic and dynamical point of view.

Centre de Recerca Matemàtica, Bellaterra, Barcelona

September 9 to 14, 2013

List of Speakers

Alexey Bolsinov, University Loughborough Leicestershire
Singularities of bi-Hamiltonian Systems and Stability Analysis

Juan José Morales-Ruiz, Universidad Politécnica de Madrid
Integrable Systems and Differential Galois Theory

Nguyen Tien Zun, Université Paul Sabatier
Geometry of Integrable non-Hamiltonian Systems

Organising and scientific committee

Vladimir Matveev, Friedrich-Schiller Universität Jena
Eva Miranda, Universitat Politècnica de Catalunya
Francisco Presas, Instituto de Ciencias Matemáticas
Iskander Taimanov, Novosibirsk State University

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CONTENTS

1. Practical Information	5
2. Timetable.....	9
3. Abstracts	11
Vladimir Matveev, Eva Miranda and Francisco Presas	11
<i>Opening and introductory talk</i>	
Roger Casals.....	11
<i>Non-trivial homotopy for contact transformations of the sphere</i>	
Chiara Esposito	11
<i>Deformation quantization of momentum map in Poisson geometry and rigidity</i>	
Sonja Hohloch	11
<i>From semi-toric systems to Hamiltonian S^1-spaces</i>	
Jiang Kai	12
<i>Some examples of geometric equivalence for non-Hamiltonian integrable systems</i>	
Stefan Rosemann.....	12
<i>c-projective geometry and its relation to integrable systems</i>	
Romero B. Solha	13
<i>On a Poincaré lemma for foliations</i>	
4. List of participants	15

1. PRACTICAL INFORMATION

You can check the updated programme at:

<http://www.crm.cat/en/Activities/Documents/TimetableADVANCED%20COURSE.pdf>

Registration: Registration to the activity will take place at the CRM, located in the Science Building (Edifici de Ciències), Universitat Autònoma de Barcelona in Bellaterra. You can check our location at: <http://www.crm.cat/en/AboutTheCRM/Pages/LocationDirections.aspx>

Lecture room: The activity will take place in the CRM Auditorium, located in the Sciences Building (Edifici de Ciències), Universitat Autònoma de Barcelona in Bellaterra.

Lodging grants: Those participants who obtained a lodging grant at the vila universitària please check the following information.

The Vila Universitària address is:

Vila Universitària, Campus UAB (go to the reception desk)
08193 Bellaterra

<http://www.uab.cat/servlet/Satellite/vila-universitaria-home-1241074134636.html>

Phone: 34-935817137

Security guard: 93 581 73 13.

marta.roig.poyato@uab.cat

You can pick up the key to your apartment at the Vila Universitària Office. They have a 24 hours emergency service after office hours, holidays, and weekends, nevertheless whenever possible you should try to arrive during office hours to avoid last minute problems. If you arrive after-hours and the guard is not in the office, you can walk to the Hotel Campus reception desk and ask them to locate the guard for you.

You can check the location at: <http://www.uab.cat/servlet/Satellite/maps-and-directions-1296719301438.html>

The CRM is located in building C (Edifici Ciències) on the “Eix central” of the UAB campus and our timetable is from 8 am to 5 pm. As for arriving to the CRM, please follow the instructions on our web page at the address: <http://www.crm.cat/en/AboutTheCRM/Pages/LocationDirections.aspx>

You can also check the link to the campus map where you will be able to locate the CRM and other facilities: <http://maps.google.es/maps/ms?ie=UTF8&hl=es&msa=0&msid=100167963947567188865.000462b5da2995f09536a&z=15>

Activity documents: Attendance certificates and registration fee receipts will be available to be picked up at the activities coordinator desk on **Friday morning**.

*Invoice: in case you need an invoice with the details of your university, please send me an email asking for it (include the information you need to be written on it).

Secretariat: The Secretariat of the CRM will be available to the participants Monday through Friday from 9:30 am to 13:30 pm.

SAF (gym at the UAB–Servei d’Activitat Física): If you have booked an apartment at the Vila Universitària we inform you that you can have free access to the SAF. Please ask at the Vila reception desk to know the conditions. They will then prepare a certificate for you in order to have free access.

Computer facilities: The computer space of the CRM will be available for the participants of the course.

The timetable is Monday through Friday from 8:30 am to 6:00 pm. The CRM premises as well as most of the UAB campus have wireless access.

Wifi password: crmwifikey

Library: The library of the Science Building of the UAB will be open from 8:30 am to 7:30 pm on working days.

Breaks: Coffee and cookies will be served during the morning breaks to all participants.

Social events: We have organized a guided visit to Barcelona Gothic quarter on Wednesday, September 11th (you should have received an email with information) and a social dinner in Barcelona on Thursday, September 12th. Registration for the dinner will be necessary before Tuesday, September 10th at noon by signing a document which will be at the auditorium’s entrance door for that purpose.

Picture: A group picture will be taken on Thursday, September 12th before the coffee break. We will inform you of the place to meet. The picture will be posted on the activity’s webpage. Questionnaire: Following

Questionnaire: Following the directions of the CRM Governing Board, we give a questionnaire to all the people participating in activities at the CRM in order to assess their level of satisfaction. The questionnaire is anonymous and not mandatory, but we would greatly appreciate it if you could answer the questions. Thank you for your cooperation.

Local emergency numbers: General emergency (police, ambulance, fire-fighters) call 112.

Safety in Barcelona: Although Barcelona is a safe city, please be aware that there is a problem with pickpockets, especially around tourist areas: La Rambla, Plaça Catalunya, Barcelona Airport, major metro and train stations, famous buildings, etc. Be sure to keep your belongings with you at all times, be alert, and be wary of unusual situations.

2. TIMETABLE

Monday, September 9	
09:15 – 09:30	REGISTRATION
09:30 – 10:30	Speakers: Vladimir Matveev, Eva Miranda and Francisco Presas <i>Opening and introductory talk</i>
10:30 – 11:00	COFFEE BREAK
11:00 – 12:00	Juan Jose Morales-Ruiz <i>Integrable systems and differential Galois theory</i>
12:15 – 13:15	Alexey Bolsinov <i>Singularities of bi-Hamiltonian systems and stability analysis</i>
13:15 – 15:00	LUNCH
15:00 – 16:00	Nguyen Tien Zung <i>Geometry of integrable non-Hamiltonian systems</i>
16:00 – 16:30	COFFEE BREAK
16:30 – 17:30	Sonja Hohloch <i>From semi-toric systems to Hamiltonian S^1-spaces</i>
Tuesday, September 10	
09:30 – 10:25	Alexey Bolsinov <i>Singularities of bi-Hamiltonian systems and stability analysis</i>
10:30 – 11:00	COFFEE BREAK
11:00 – 12:00	Juan Jose Morales-Ruiz <i>Integrable systems and differential Galois theory</i>
12:15 – 13:15	Romero B. Solha <i>On a Poincaré lemma for foliations</i>
13:15 – 15:00	LUNCH
15:00 – 16:00	Nguyen Tien Zung <i>Geometry of integrable non-Hamiltonian systems</i>
16:00 – 16:30	COFFEE BREAK
16:30 – 17:30	Jiang Kai <i>Some examples of geometric equivalence for non-Hamiltonian integrable systems</i>
Wednesday, September 11	
	FREE DAY. GUIDED VISIT

Thursday, September 12	
09:30 – 10:30	Juan Jose Morales-Ruiz <i>Integrable systems and differential Galois theory</i>
10:30 – 11:00	COFFEE BREAK
11:00 – 12:00	Nguyen Tien Zung <i>Geometry of integrable non-Hamiltonian systems</i>
12:15 – 13:15	Chiara Esposito <i>Deformation quantization of momentum map in Poisson geometry and rigidity</i>
13:15 – 15:00	LUNCH
15:00 – 17:30	OPEN PROBLEMS SESSION (BY SPEAKERS)
	DINNER IN BARCELONA AT 20:00
Friday, September 13	
09:30 – 10:30	Alexey Bolsinov <i>Singularities of bi-Hamiltonian systems and stability analysis</i>
10:30 – 11:00	COFFEE BREAK
11:00 – 12:00	Nguyen Tien Zung <i>Geometry of integrable non-Hamiltonian systems</i>
12:15 – 13:15	Roger Casals <i>Non-trivial homotopy for contact transformations of the sphere</i>
13:15 – 15:00	LUNCH
15:00 – 17:30	OPEN PROBLEMS SESSION (BY ORGANIZERS)
Saturday, September 14	
09:30 – 10:30	Alexey Bolsinov <i>Singularities of bi-Hamiltonian systems and stability analysis</i>
10:30 – 10:45	BREAK
10:45 – 11:45	Juan Jose Morales-Ruiz <i>Integrable systems and differential Galois theory</i>
12:00 – 13:00	Stefan Rosemann <i>c-projective geometry and its relation to integrable systems</i>

3. ABSTRACTS

Vladimir Matveev, Eva Miranda and Francisco Presas

Opening and introductory talk.

Abstract: We will open the special CRM Research Program on *Geometry and Dynamics on Integrable Systems* and we will present the main objects of study of the three minicourses of the Advanced Course in a unified way.

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Roger Casals

Non-trivial homotopy for contact transformations of the sphere.

Abstract: We discuss the non-triviality of the Reeb flow for the $(2n + 1)$ -dimensional standard contact spheres inside the fundamental group of their contactomorphism group, n greater than 3. Higher-homotopy of this contactomorphism group will also be treated.

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Chiara Esposito

Deformation quantization of momentum map in Poisson geometry and rigidity.

Abstract: In this talk, we will recall the notion of a momentum map associated to actions of Poisson Lie groups on Poisson manifolds and discuss the rigidity of such a map as an application of a normal form theorem. This result generalizes the rigidity of the momentum map in the canonical setting, which will be recalled and we will sketch the proof.

We will define the quantization of the Poisson action and the momentum map using the approach of formal deformation quantization and the quantum groups. Finally, we will introduce the problem of the rigidity of the quantum momentum map, also in terms of normal form theorem.

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Sonja Hohloch

From semi-toric systems to Hamiltonian S^1 -spaces.

Abstract: This paper studies the local and global aspects of semi-toric integrable systems, introduced by Vũ Ngọc, using ideas stemming from the theory of Hamiltonian S^1 -spaces developed by Karshon. First, we show how any labeled convex polygon associated to a semi-toric system (as defined by Vũ Ngọc) determines Karshon's labeled directed graph which classifies the underlying Hamiltonian S^1 -space up to isomorphism. Then we characterize adaptable semi-toric systems,

i.e. those whose underlying Hamiltonian S^1 -action can be extended to an effective Hamiltonian \mathbb{T}^2 -action, as those which have at least one associated convex polygon which satisfies the Delzant condition.

This is a joint work with Silvia Sabatini and Daniele Sepe.

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Jiang Kai

Some examples of geometric equivalence for non-Hamiltonian integrable systems.

Abstract: Nguyen Tien Zung gave a new notion between two completely integrable systems in the viewpoint of geometric aspect, which is called *geometrically equivalent*. Zung has proved that analytic nondegenerate system and its linear part are geometrically equivalent and also two special cases in the smooth category.

I will talk about some examples of his notion in the smooth category. When we have several vector fields of certain type and their common first integrals, we could manage to put them into Poincaré-Dulac normal forms simultaneously, which implies the geometric equivalence.

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Stefan Rosemann

c-projective geometry and its relation to integrable systems.

Abstract: Two Kähler metrics are called *c-projectively equivalent* if their J -planar curves coincide. These curves are defined by the property that the acceleration is complex proportional to the velocity.

If a Kähler metric admits a *c-projectively equivalent* metric, it does admit a family of mutually commuting integrals of first and second order for the geodesic flow of the metric (i.e. a family of commuting Killing vectors and Killing 2-tensors) which, under some additional assumption, leads to Liouville integrability of the geodesic flow of the metric.

The goal of this talk is to explain the relation between *c-projectively equivalent* metrics and integrable geodesic flows in more detail. In particular, I will explain how the integrals helped to obtain a complete local and global classification of Kähler metrics that do admit *c-projectively equivalent* metrics which, in turn, provides a lot of new examples of integrable systems on Kähler manifolds.

This part of the talk recalls the results of Topalov, Kiyohara and Apostolov et al who did the classification when the signature of the metric is Riemannian. Using the integrals, I will show how to obtain the classification when the signature is arbitrary. This has been done in a recent joint work with Bolsinov, Matveev and Mettler.

As another application of the integrals appearing in c-projective geometry, I will show how they helped to give the proof of the classical Yano-Obata conjecture which was obtained in joint work with Matveev

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Romero B. Solha

On a Poincaré lemma for foliations.

Abstract: I am going to discuss the (non)existence of a Poincaré lemma for foliated cohomology when the foliation is singular. The talk will present some results obtained with Eva Miranda in the particular case of foliations induced by nondegenerate integrable systems on symplectic manifolds, and their relationship with the theory of Geometric Quantisation.

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4. LIST OF PARTICIPANTS

Name	Institution
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Alexei Bolsinov	University Loughborough Leicestershire
Roger Casals	ICMAT/CSIC Madrid
Alvaro del Pino	ICMAT/CSIC Madrid
David Dowell	
Viviana A. Díaz	Universidad Nacional del Sur, Buenos Aires
Chiara Esposito	Mathematisches Forschungsinstitut Oberwolfach
Pedro Frejlich	University Utrecht
Tamás F. Görbe	University of Szeged
Sonja Hohloch	École Polytechnique Fédérale de Lausanne
Kai Jiang	
Michael Khanevsky	
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