Abstracts of the Speakers

Alain Albouy

On Newtonian dynamics and Beltrami's theorem on projectively flat Riemannian manifolds.

Abstract: Appell extended in 1891 the theory of the central projection from geometry to Newtonian dynamics. He noticed in 1892 that Beltrami's theorem constrains the interesting target spaces for the central projection. We will develop his remark by showing that his theory also suggests an extension and a new proof of Beltrami's theorem. The extension concerns degenerate inner products.

Contact address: `albouy@imcce.fr`

Vincent Colin

Higher-dimensional Heegard Floer homology.

Abstract: In a work in progress with Ko Honda, we extend the definition of the hat version of Heegaard Floer homology to contact manifolds of arbitrary odd dimension using higher-dimensional open book decompositions and the theory of Weinstein domains. This also suggests a reformulation and an extension of Symplectic Khovanov homology to links in arbitrary 3-manifolds.

Contact address: `vincent.colin@univ-nantes.fr`

Lucia Di Vizio

Difference Galois theory of differential equations.

Abstract: I'll explain how one can construct a Galois theory for differential equations that takes into account the action of a difference operator,i.e., an endomorphisms, on the solutions. The theory attaches to a linear differential equation a group scheme, which encodes the algebraic difference relations among the solutions of the differential equation. This is typically the case of p-adic differential equation with a Frobenius structure. As an application, I'll give a Galosian characterization of "discrete isomonodromy".

This is a joint work with C. Hardouin and M. Wibmer.

Contact address: `Lucia.Di.Vizio@math.cnrs.fr`

Christian Duval
Quantum integrability and conformally equivariant quantization.

Abstract: I will introduce a special Liouville-integrable system, namely the dual Jacobi-Moser system, associated with the geodesic flow on the n-sphere endowed with a conformally flat metric projectively equivalent to that of the generic n-ellipsoid. I will go on proving that quantum integrability of both dual Jacobi-Moser and Neumann-Uhlenbeck systems is actually ensured by means of conformally equivariant quantization.

This is joint work with Galliano Valent.

Contact address: `christian.duval@cpt.univ-mrs.fr`

Rui Loja Fernandes
Global geometry of non-commutative integrable systems.

Abstract: I will describe the global geometry of NCIS on Poisson manifolds, where twisted Dirac structures appear naturally. As an application, we give the obstructions to the existence of global action-angle variables.

Contact address: `ruiloja@illinois.edu`

Valerij Vasilievich Kozlov
Polynomial in momenta integrals for systems of interacting particles in a box with ellastic walls.

Abstract: We discuss conditions for the existence of an additional to the energy, polynomial in momenta first integral in a system of interacting particles, which move in a multidimensional parallelepiped, ellastically reflecting from its faces. One of standard examples for such a system is the Boltzmann-Gibbs gas i.e., the system of small balls in a rectangular parallelepiped which collide ellastically with each other and with walls of the box. An old (and still open) conjecture says that the Boltzmann-Gibbs system is ergodic provided the dimension is greater than one. If the particles attract each other, apparently, one should not expect ergodicity. Absence of additional integrals is a weaker property in comparison with ergodicity. However results on polynomial integrals give some progress in the problem of statistical justification of thermodynamics.

Contact address: `kozlov@pran.ru`

Boris Kruglikov

Einstein-Weyl geometry and Integrability of dispersionless PDEs.

Abstract: In the joint work with Eugene Ferapontov we showed that for 2nd order PDE in 3D integrability by the method of hydrodynamic reduction is equivalent to the Einstein-Weyl property of the symbol of the equation. Thus linearization of an integrable equation carries an integrable geometry. I will illustrate this with many examples, relating the Einstein-Weyl property to the existence of Lax pairs and to the finite type geometry of 3rd order ODEs.

Contact address: `boris.kruglikov@uit.no`

Ivan Sergeevich Mamaev

Absolute dynamics of the Chaplygin ball: topological and analytical methods.

Abstract: In this work we consider the classical problem of a balanced dynamically asymmetric ball rolling on a plane without slipping. As is well known [1, 2, 3], in the nonholonomic setting the equations of motion for this system can be written as

$$\dot{\boldsymbol{M}} = \boldsymbol{M} \times \boldsymbol{\omega}, \quad \dot{\boldsymbol{\gamma}} = \boldsymbol{\gamma} \times \boldsymbol{\omega}, \quad \dot{\boldsymbol{\alpha}} = \boldsymbol{\alpha} \times \boldsymbol{\omega}, \quad \dot{\boldsymbol{\beta}} = \boldsymbol{\beta} \times \boldsymbol{\omega},$$
$$\dot{x} = b(\boldsymbol{\omega}, \boldsymbol{\beta}), \quad \dot{y} = -b(\boldsymbol{\omega}, \boldsymbol{\alpha}),$$

where $\boldsymbol{M}$ is the angular momentum of the ball relative to the point of contact, α, β and γ are the unit vectors of a fixed coordinate system, written in projections onto the principal axes of inertia of the ball, x and y are the coordinates of the center of the ball in absolute space, b is the radius of the ball, and ω is the angular velocity of the ball, which is expressed in terms of the angular momentum by the relation

$$\boldsymbol{\omega} = \mathbf{A}(\boldsymbol{M} + Z\boldsymbol{\gamma}),$$
$$\mathbf{A} = (\mathbf{I} + \mathcal{D}\mathbf{E})^{-1}, \quad Z = \frac{(\mathbf{A}\boldsymbol{M}, \boldsymbol{\gamma})}{\mathcal{D}^{-1} - (\boldsymbol{\gamma}, \mathbf{A}\boldsymbol{\gamma})}.$$

Here $\mathcal{D} = mb^2$, and $\mathbf{I} = \operatorname{diag}(\mathrm{I}_1, \mathrm{I}_2, \mathrm{I}_3)$ and m are the tensor of inertia and the mass of the ball, respectively.

In this case the equations for the orientation of the ball decouple and can be integrated separately. After that for the given law of motion, the position of the point of contact is found by quadratures. Thus, our main goal is to analyze the behavior of the point of contact depending on the evolution of the orientation of the ball. In particular, using topological methods, we identify the region in the space of first integrals of the system, in which a strictly positive drift is observed in the direction perpendicular to the projection of the angular momentum vector onto the plane. In addition, we use analytical methods to show that for almost all initial conditions there is no drift along the projection of the angular momentum vector onto the plane.

This is a joint work with Alexey Borisov, Alexander Kilin.

References

[1] Chaplygin, S. A., *On a Ball's Rolling on a Horizontal Plane*, Math. Sb., 1903, vol. 24, no. 1, pp. 139–168 [Regul. Chaotic Dyn., 2002, vol. 7, no. 2, pp. 131–148].
[2] Duistermaat J. J. *Chaplygin's Sphere*. arXiv:math/0409019v1
[3] Kilin A. A. *The Dynamics of Chaplygin Ball: the Qualitative and Computer Analysis*, Regul. Chaotic Dyn., 2001, vol. 6, no. 3, pp. 291–306.

Contact address: mamaev@rcd.ru

Jean-Pierre Marco
Polynomial entropy, integrability and geodesic flows.

Abstract: We will first introduce the notion of polynomial entropy and show several examples for which explicit computations are possible: homeomorphisms of the circle, vector fields on the torus, action-angle Hamiltonian systems, Hamiltonian and gradient vector fields on surfaces. We will then examine various notions of integrability (Bott, Williamson, C^0) together with their related entropic properties. We will finally discuss recent progress by Labrousse in the characterization of flat metrics on the two dimensional torus by the minimization of the polynomial entropy of the associated geodesic flows (the "fla" analogue of the Katok and Besson-Courtois-Gallot theorem in hyperbolic geometry).

Contact address: marco.jeanpierre@gmail.com

Andrey Mironov
Semi-Hamiltonian system for integrable geodesic flows on 2-torus.

Abstract: We prove that the question of existence of polynomial first integrals of the geodesic flow on 2-torus leads to a semi-Hamiltonian quasi-linear equations, i.e. the system can be written in the conservation lows form and in the hyperbolic region it has Riemannian invariants. We also prove that in the elliptic region cubic and quartic integrals are reduced to the integrals of degree one or two. The results obtained with Misha Bialy.

Contact address: mironov@math.nsc.ru

Klaus Niederkrueger
Understanding subcritical surgeries with holomorphic curves.

Abstract: A common technique of modifying contact manifolds consists in using contact surgery. The aim of our work (in progress) is to show that the belt sphere of a subcritical surgery is contractible in every symplectically aspherical filling. This is a joint work with P. Ghiggini and C. Wendl.

Contact address: niederkr@math.univ-toulouse.fr

Valentin Ovsienko
Pentagram map and frieze patterns.

Abstract: The pentagram map is a discrete completely integrable system whose continuous limit is the Boussinesq equation. It is closely related to the theory of cluster algebras. The pentagram map and its analogs act on interesting and complicated spaces. The simplest of them is the classical moduli space $M_{0,n}$ of rational curves of genus 0. These moduli spaces have a rich combinatorial structure related to the notion of "Coxeter frieze pattern" and can be understood as a "cluster manifolds".

Contact address: `ovsienko@math.univ-lyon1.fr`

Daniel Peralta
Integrability in three-dimensional steady fluid flows.

Abstract: I will talk about some integrability results of steady solutions to the Euler equation. In particular, I will review the celebrated Arnold's structure theorem and its generalizations, and show some extensions to Beltrami flows with nonconstant proportionality factor.

Contact address: `dperalta@icmat.es`

Konrad Schöbel
Separation coordinates and moduli spaces of stable curves.

Abstract: We establish a surprising link between two a priori completely unrelated objects: The space of isometry classes of separation coordinates for the Hamilton-Jacobi equation on an n-dimensional sphere and the Deligne-Mumford moduli space $\bar{\mathcal{M}}_{0,n+2}$ of stable algebraic curves of genus zero with $n+2$ marked points. We use the rich combinatorial structure of the latter and the closely related Stasheff polytopes in order to classify the different canonical forms of separation coordinates. Moreover, we infer an explicit construction for separation coordinates and the corresponding quadratic integrals from the mosaic operad on $\bar{\mathcal{M}}_{0,n+2}$.

This is a joint work with Alexander P. Veselov.

Contact address: `konrad.schoebel@uni-jena.de`

Dmitry Treschev
Billiard map and rigid rotation.

Abstract: Can a billiard map be locally conjugated to a rigid rotation? We prove that the answer to this question is positive in the category of formal series. We also present numerical evidence that for "good" rotation angles the answer is also positive in analytic category

Contact address: `treschev@mi.ras.ru`

Jacques-Arthur Weil

Variational approach to the irreducibility of order two non linear differential equations.

Abstract: In his work on the first Painlevé equation, Guy Casale gave a characterization of the reducibility of non linear differential equations using Malgrange pseudogroup methods and classifications of E. Cartan. In our joint work, we propose an irreducibility criterion for second order non linear differential equations, based on the dimension of their Malgrange pseudogroup. We give a method to measure lower bounds on this dimension and apply this to the irreducibility of a Painlevé II equation.

This is a joint work with Guy Casale.

Contact address: `jacques-arthur.weil@unilim.fr`

Abstracts of the Contributed Talks

Ángel Ballesteros

Integrability and deformations of Lotka-Volterra systems from Poisson-Lie dynamics.

Abstract: The integrability properties of a class of three-dimensional (3D) Lotka-Volterra equations are revisited from a novel point of view by showing that the quadratic Poisson structure that provides the Hamiltonian structure of the system can be interpreted as a multiparameter real three-dimensional Poisson-Lie group. As a consequence, by considering the most generic Poisson-Lie structure on this family of Lie groups, a new two-parametric integrable perturbation of the 3D LV system through polynomial and rational perturbation terms is explicitly found. Moreover, the Poisson coalgebra map that is de

ned by the group multiplication provides the keystone for the explicit construction of a new family of 3N-dimensional integrable systems with coalgebra symmetry that, under certain conditions, contains N nested sets of deformed versions of the 3D Lotka-Volterra equations.

This is a joint work with Alfonso Blasco and Fabio Musso.

References

[1] A. Ballesteros, A. Blasco, F.J. Herranz, F. Musso, O. Ragnisco, *(Super)integrability from coalgebra symmetry: formalism and applications*, J. Phys. Conf. Series **175**, 012004 (26 pp) (2009).

[2] A. Ballesteros, A. Blasco, F. Musso, *Integrable deformations of Lotka-Volterra systems*, Phys. Lett. A **375** (2011), 3370–3374.

[3] A. Ballesteros, A. Blasco, F. Musso, *Non-coboundary Poisson-Lie structures on the book group*, J. Phys. A: Math. Theor. **45** (2012), 105205 (14 pp).

Contact address: `angelb@ubu.es`

Thomas Dreyfus

Density theorem for parameterized differential Galois theory.

Abstract: To a linear differential system with coefficients that are germs of meromorphic functions, we can associate an algebraic group (the differential Galois group), who measures the algebraic relations between the solutions. The density Theorem of Ramis gives a list of topological generators of this group, for Zariski topology. More recently has been developed by Cassidy and Singer a Galois theory for parameterized linear differential system. This time, the Galois group, which is a differential group, measure the algebraic and the differential (with respect to the parameters) relations between the solutions. We will present an analogue of the density theorem of Ramis for this theory.

Contact address:

Yuri Fedorov

.

Abstract:

Contact address:

Laszlo Feher

New applications of the reduction approach to integrable many-body systems.

Abstract: Many important integrable Hamiltonian systems can befruitfully realized as reductions of "canonical free systems" having rich symmetries on higher dimensional phase spaces. We first review the results of the last few years concerning the application of the reduction method towards explaining the duality relations of integrable many-body systems of Ruijsenaars-Schneider type in group theoretic terms. We then present fresh results such as the construction of new compact forms of the trigonometric Ruijsenaars-Schneider system by quasi-Hamiltonian reduction and a novel group theoretic interpretation of the action-angle map of the standard open Toda lattice.

Contact address: `lfeher@physx.u-szeged.hu`

Andriy Panasyuk

Lie pencils on $\mathfrak{sl}(n)$ ***and integrable geodesic flows.***

Abstract: A Lie bracket on $\mathfrak{sl}(n,\mathbb{C})$ compatible with the standard commutator will be described. Using this bracket and the standard techniques from the theory of Lie–Poisson pencils one builds complete families of functions in involution which can serve as integrals of geodesic flows on related compact Lie groups.

This is a joint work with Krzysztof Smiarowski.

Contact address: `panas@matman.uwm.edu.pl`

Vladimir Salnikov

Graded geometry in gauge theories and beyond.

Abstract: We study some graded geometric constructions appearing naturally in the con- text of gauge theories. We introduce the language of Q-bundles convenient for description of the symmetries of sigma models. Inspired by a known relation of gauging with equivariant cohomology we generalize the latter notion to the case of arbitrary Q-manifolds introducing thus the concept of equivariant Q-cohomology.

As the main examples of application of these constructions we consider the Poisson sigma model, which is related to the derivation of the famous Kontsevich quantization formula, the G/G Wess-Zumino-Witten model, as well as their common generalization –the Dirac sigma model. We obtain these models by a gauging-type procedure of the action of a group related to n-plectic manifolds and describe their symmetries in terms of classical differential geometry. We also recover the closure (or the anomaly cancellation) property of the group of symmetries as the integrability condition present in the context of Dirac structures.

We comment on other possible applications of the suggested approach including the analysis of supersymmetric gauge theories.

This is a joint work with Thomas Strobl.

Contact address: vladimir.n.salnikov@gmail.com

Bela Gabor Pusztai

The hyperbolic $BC(n)$ ***Sutherland and the rational*** $BC(n)$ ***Ruijsenaars-Schneider-van Diejen models: Lax matrices, duality and scattering theory.***

Abstract: In this talk we wish to report on our recent results on the hyperbolic $BC(n)$ Sutherland and the rational $BC(n)$ Ruijsenaars-Schneider-van Diejen models. After explaining the recently established action-angle duality between these integrable particle systems, we briefly discuss their scattering theory as well.

Contact address: gpusztai@math.u-szeged.hu