New periodic solutions of Lennard–Jones* $2-$ *and* $3-$*body problems and related abstract results

Daniel Strzelecki, Nicolaus Copernicus University in Torun
E-mail address: danio@mat.umk.pl.

The aim of the talk is to show the existence of new families of periodic solutions in Lennard-Jones $2-$ and $3-$ body problems. This results will be a consequences of two symmetric versions of Liapunov center theorem for Newtonian systems which has been proved in [2, 3]. Moreover, the new theorem for study Hamiltonian systems with symmetries will be shown.

Define Lennard-Jones potential $U\colon \Omega \to \mathbb{R}$ by formula

$$U(q) = \sum_{1 \leq i < j \leq N} \left(\frac{1}{\mid q_i - q_j \mid^{12}} - \frac{2}{\mid q_i - q_j \mid^{6}} \right).$$

- 2-body problem

  For $N = 2$ it was shown in [1] that $(\nabla U)^{-1}(0) \cap \Omega = \{(q_1, q_2) \in \Omega : q_1 = -q_2 \text{ and } |q_1 - q_2| = 1\}$ and this circle consists of minima of potential U. We prove the existence of new periodic solutions of the problem in any neighborhood of this circle with minimal periods close to $\frac{\pi}{6}$.

- 3-body problem

  In the 3-body problem we know five circular families of stationary solutions of the problem (see [1]). Two of them are non-colinear and form a circles of critical points. We prove the existence of two new families of non-stationary periodic solutions of the problem in any neighborhood of these oribts, estimating minimal periods of these families by $\frac{\pi}{3\sqrt{3}}$ and $\frac{\pi}{3\sqrt{6}}$.

References

[1] M. Corbera, J. Llibre, and E. Pérez-Chavela., Equilibrium points and central configurations for the Lennard- Jones 2- and 3-body problems. Celestial Mech. Dynam. Astronom. 89 (2004), no. 3, 235–266.

[2] E. Pérez-Chavela, S. Rybicki, and D. Strzelecki, Symmetric Liapunov center theorem. Calc. Var. Partial Differential Equations 56 (2017), no. 2, 26.

[3] E. Pérez-Chavela, S. Rybicki, and D. Strzelecki, Symmetric Liapunov center theorem for minimal orbit. J. Differential Equations 265 (2018), no. 3, 752–778.