Transversal connecting orbits of singularly perturbed Lagrangian systems with turning points

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We study a singlularly perturbed Lagrangian system with Lagrangian

$$L(q, \dot{q}, t, \varepsilon) = \frac{\varepsilon^2}{2}|\dot{q}|^2 - f(t)V(q), \;\; \varepsilon \ll 1 \tag{1}$$

defined on a compact Riemannian manifold $\mathcal{M}$. It is assumed the potential V is of class $C^2(\mathcal{M})$, f is periodic with period 1 and the system (1) has M turning points, i.e.
(A_1) there exist M different solutions $t_l \in \mathbb{T}, l = 1, \ldots, M$ of the equation $f(t) = 0$;
(A_2) for each $l = 1, \ldots, M$ there exists a neighborhood of t_l where f can be represented as $f(t) = (t - t_l)^{\kappa_l} g_l(t)$ with $\kappa_l \in \mathbb{N}$ and some C^1-function g_l such that $g_l(t_l) \neq 0$.

Let X_c denotes a subset of $\mathcal{M}$ at which $V(x)$ distinguishes its maximum or minimum. We assume that
(A_3) X_c consists of isolated nondegenerate critical points of V.
In a vicinity of a turning point the system (1) can be approximated by the reference system with Lagrangian

$$L(q, q', \zeta) = \frac{1}{2}|q'|^2 - \zeta^\kappa V(q), \quad \kappa \in \mathbb{N}, \quad q' = \frac{\mathrm{d}q}{\mathrm{d}\zeta}. \tag{2}$$

Using variational approach it is proved the existence of infinitely many connecting trajectories for the reference system (2). We also apply the Newton-Kantorovich method to establish sufficient conditions for transversality of such trajectories.

Under transversality assumption on the connecting orbits of the reference system we proved the existence of doubly asymptotic trajectories for the system (1) which shadow the connecting orbits of the reference system. In particular, it was proved the following

Theorem. *For any sequence* $\{x_k\}_{k=1}^n \subset X_c$ *and any* $\rho > 0$ *there exists* $\varepsilon_0 > 0$ *and a subset* $\mathcal{E}_h \subset (0, \varepsilon_0)$ *such that*

1. *for any* $\varepsilon_1 < \varepsilon_0$ *the Lebesgue measure* $leb\left((0, \varepsilon_1) \setminus \mathcal{E}_h\right) = O\left(\mathrm{e}^{-c/\varepsilon_1}\right)$ *with some positive constant c;*

2. *for any* $\varepsilon \in \mathcal{E}_h$ *there exist infinitely many doubly asymptoic trajectories of the system* (1) *which emanate from* x_1*, terminate at* x_n *and pass through balls of radii* ρ *centred at the points* x_k *in an order induced by the chain* $(x_1, x_2, \ldots, x_n)$.

References

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