Oscillatory motions in restricted N-body problems

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We consider the planar restricted N-body problem where the $N-1$ primaries are assumed to be in a central configuration whereas the infinitesimal particle escapes to infinity in a parabolic orbit. We prove the existence of transversal intersections between the stable and unstable manifolds of the parabolic orbits at infinity which guarantee the existence of a Smale's horseshoe. This implies the occurrence of chaotic motions but also of oscillatory motions, that is, orbits for which the massless particle leaves every bounded region but it returns infinitely often to some fixed bounded region. Our achievement is based in an adequate scaling of the variables which allows us to write the Hamiltonian function as the Hamiltonian of the Kepler problem plus higher-order terms that depend on the chosen configuration. We compute the Melnikov function related to the first non-null perturbative term and characterize the cases where it has simple zeroes. Concretely, for some combinations of the configuration parameters, i.e. mass values and positions of the primaries, and for a specific value of a parameter related to the angular momentum vector, the Melnikov function vanishes, otherwise it has simple zeroes and the transversality condition is satisfied. When the Melnikov function corresponding to the principal part of the perturbation is zero we compute the next non-zero Melnikov function proving that it has simple zeroes. The theory is illustrated for various cases of restricted N-body problems, including the circular restricted three-body problem. No restrictions on the mass parameters are assumed.

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